Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.3-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [44]. This is test number [33].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (44)	0.00 (0)
Mathematica	100.00 (44)	0.00 (0)
Maple	100.00 (44)	0.00 (0)
Mupad	100.00 (44)	0.00 (0)
Fricas	95.45 (42)	4.55 (2)
Sympy	81.82 (36)	% 18.18 (8)
Giac	72.73 (32)	27.27 (12)
Maxima	27.27 (12)	72.73 (32)
IntegrateAlgebraic	0.00 (0)	100.00 (44)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

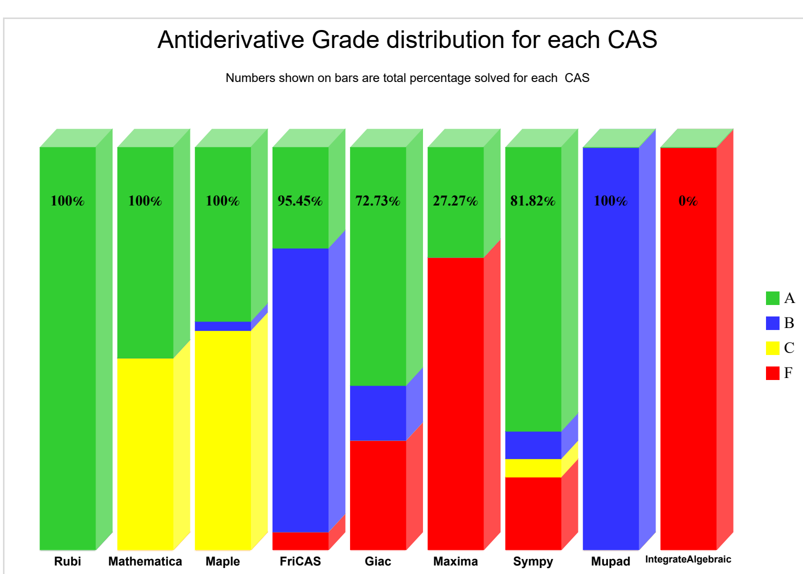
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

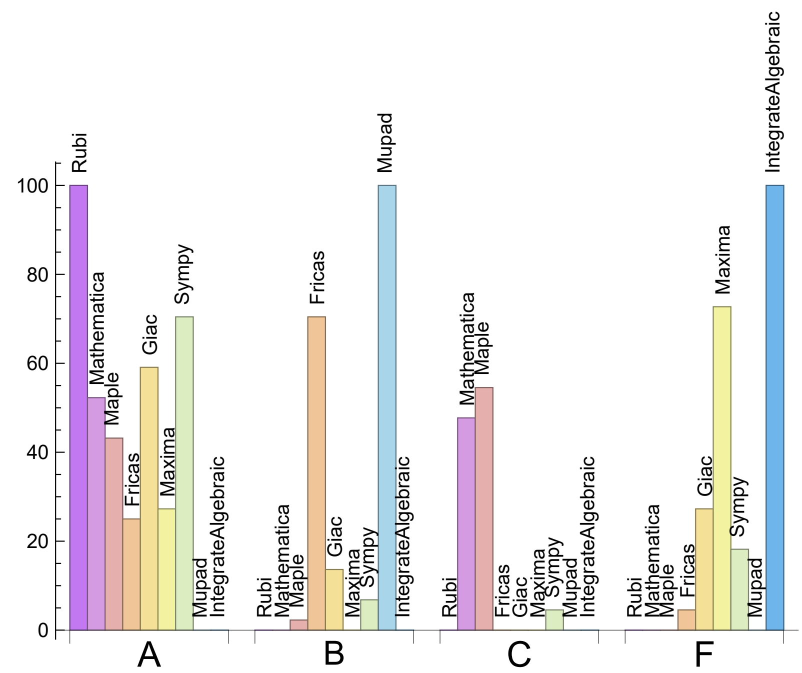
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Sympy	70.45	6.82	4.55	18.18
Giac	59.09	13.64	0.00	27.27
Mathematica	52.27	0.00	47.73	0.00
Maple	43.18	2.27	54.55	0.00
Maxima	27.27	0.00	0.00	72.73
Fricas	25.00	70.45	0.00	4.55
IntegrateAlgebraic	0.00	0.00	0.00	100.00
Mupad	N/A	100.00	0.00	0.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	2	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	44	100.00 %	0.00 %	0.00 %
Giac	12	8.33 %	33.33 %	58.33 %
Maxima	32	96.88 %	0.00 %	3.12 %
Sympy	8	0.00 %	75.00 %	25.00 %
Mupad	0	0.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

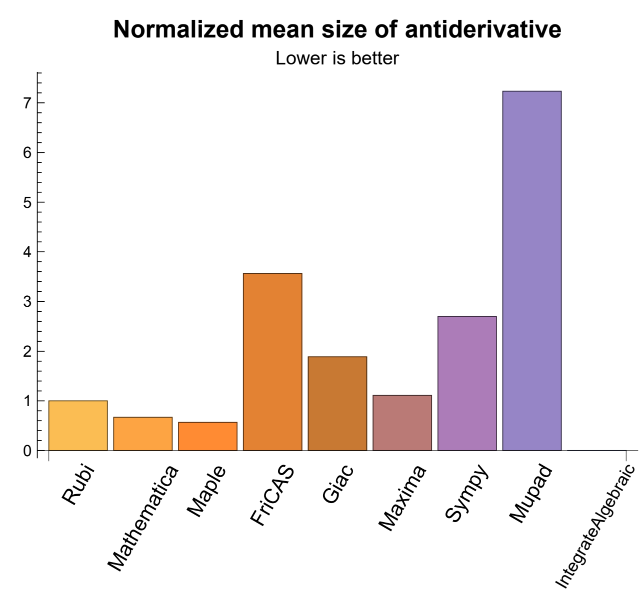
1.3 Performance

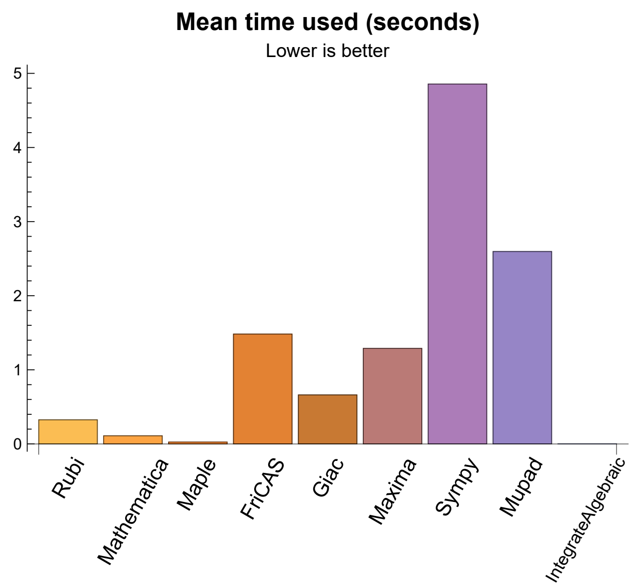
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.33	293.70	1.00	213.00	1.00
Mathematica	0.11	143.20	0.67	87.00	0.74
Maple	0.03	100.00	0.57	55.00	0.35
Maxima	1.29	170.50	1.11	145.00	0.96
Fricas	1.48	1164.67	3.57	560.00	2.52
Sympy	4.86	442.97	2.70	75.50	0.40
Giac	0.66	383.88	1.89	215.00	0.91
Mupad	2.60	3133.18	7.23	355.50	2.02
IntegrateAlgebraic	0.00	0.00	0.00	0.00	0.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{12, 23\}$

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

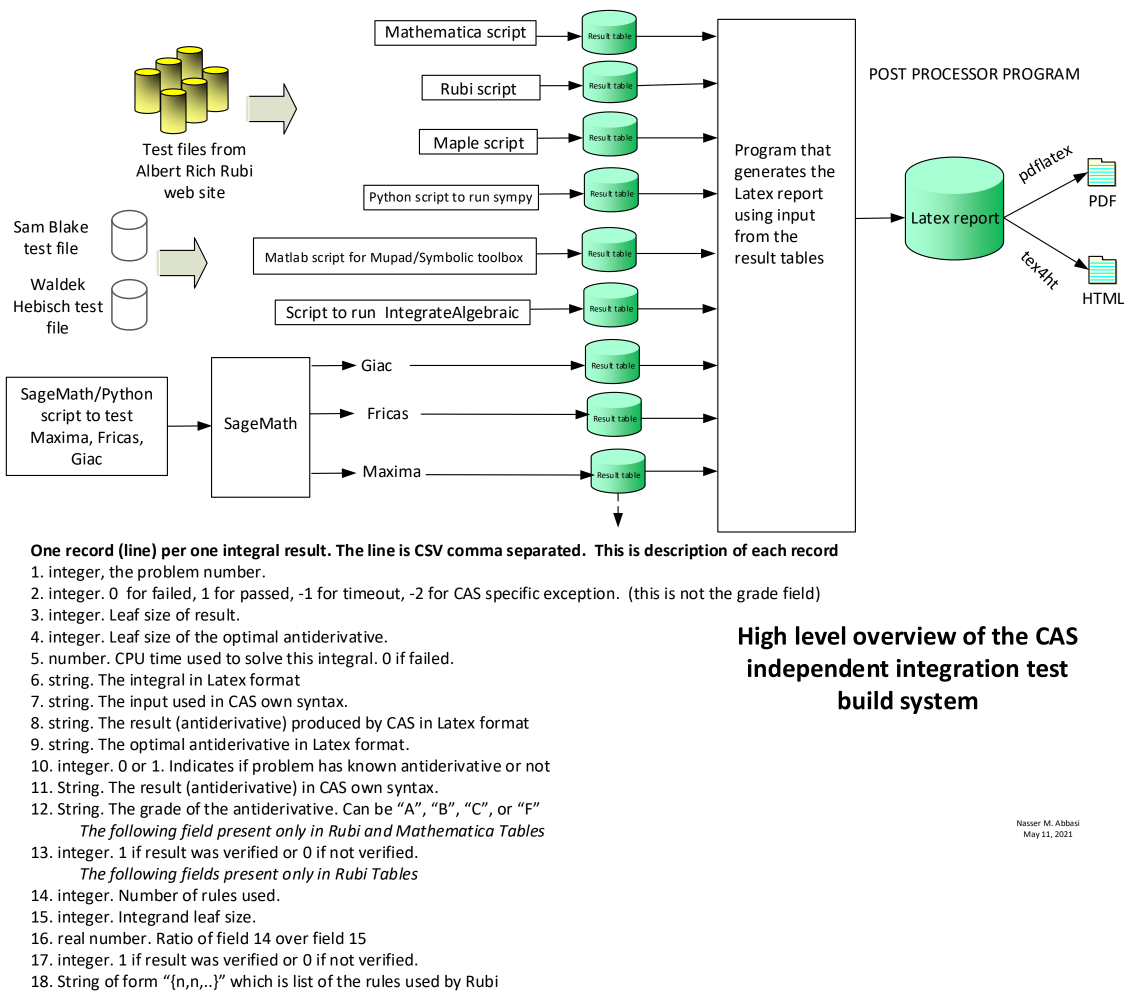
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 11, 13, 15, 16, 19, 22, 24, 26, 27, 30, 34, 35, 36, 37, 38, 40, 42, 43, 44 }

B grade: { }

C grade: { 5, 6, 7, 8, 9, 10, 12, 14, 17, 18, 20, 21, 23, 25, 28, 29, 31, 32, 33, 39, 41 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 11, 12, 15, 16, 19, 22, 23, 26, 27, 30, 34, 35, 36, 38, 42, 43, 44 }

B grade: { 37 }

C grade: { 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 17, 18, 20, 21, 24, 25, 28, 29, 31, 32, 33, 39, 40, 41 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 11, 15, 22, 26, 34, 36, 38, 42, 43, 44 }

B grade: { }

C grade: { }

F grade: { 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 40, 41 }

2.1.5 FriCAS

A grade: { 11, 12, 14, 22, 23, 26, 31, 32, 33, 34, 35 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 24, 25, 27, 28, 29, 30, 36, 37, 38, 40, 42, 43, 44 }

C grade: { }

F grade: { 39, 41 }

2.1.6 Sympy

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 28, 29, 30, 31, 36, 38, 42, 43, 44 }

B grade: { 26, 34, 35 }

C grade: { 12, 23 }

F grade: { 3, 4, 32, 33, 37, 39, 40, 41 }

2.1.7 Giac

A grade: { 1, 2, 3, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 27, 30, 31, 32, 33, 34, 35, 36, 38, 40 }

B grade: { 4, 26, 37, 42, 43, 44 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 17, 18, 20, 28, 29, 39, 41 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44 }

C grade: { }

F grade: { }

2.1.9 IntegrateAlgebraic

A grade: { }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	305	334	329	282	3224	165	288	1331	0
N.S.	1	1.00	1.10	1.08	0.92	10.57	0.54	0.94	4.36	0.00
time (sec)	N/A	0.249	0.100	0.115	1.494	1.625	3.111	0.429	1.542	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	323	323	337	386	313	3178	168	308	1293	0
N.S.	1	1.00	1.04	1.20	0.97	9.84	0.52	0.95	4.00	0.00
time (sec)	N/A	0.189	0.116	0.110	1.341	1.943	3.123	0.379	2.973	0.001

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	754	754	534	34	0	3406	0	601	2510	0
N.S.	1	1.00	0.71	0.05	0.00	4.52	0.00	0.80	3.33	0.00
time (sec)	N/A	1.247	0.628	0.018	0.000	2.406	0.000	0.737	2.780	0.001

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	329	425	39	0	3385	0	633	2438	0
N.S.	1	1.00	1.29	0.12	0.00	10.29	0.00	1.92	7.41	0.00
time (sec)	N/A	0.209	0.134	0.013	0.000	2.843	0.000	0.753	2.719	0.001

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	791	791	67	53	0	3059	136	0	10409	0
N.S.	1	1.00	0.08	0.07	0.00	3.87	0.17	0.00	13.16	0.00
time (sec)	N/A	0.863	0.045	0.049	0.000	1.858	8.503	0.000	3.825	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	791	791	67	53	0	3059	136	0	10411	0
N.S.	1	1.00	0.08	0.07	0.00	3.87	0.17	0.00	13.16	0.00
time (sec)	N/A	0.805	0.035	0.049	0.000	1.798	7.139	0.000	4.030	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	69	55	0	3048	136	0	10337	0
N.S.	1	1.00	0.20	0.16	0.00	8.73	0.39	0.00	29.62	0.00
time (sec)	N/A	0.422	0.045	0.033	0.000	1.706	8.251	0.000	4.035	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	751	751	69	55	0	3051	136	0	10343	0
N.S.	1	1.00	0.09	0.07	0.00	4.06	0.18	0.00	13.77	0.00
time (sec)	N/A	0.925	0.039	0.034	0.000	1.615	7.255	0.000	4.204	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	55	42	0	1443	75	0	5341	0
N.S.	1	1.00	0.13	0.10	0.00	3.51	0.18	0.00	13.00	0.00
time (sec)	N/A	0.292	0.026	0.056	0.000	1.408	3.669	0.000	3.683	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	451	451	55	42	0	951	24	239	459	0
N.S.	1	1.00	0.12	0.09	0.00	2.11	0.05	0.53	1.02	0.00
time (sec)	N/A	0.407	0.015	0.010	0.000	1.243	1.475	0.931	0.176	0.001

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	64	58	72	95	73	72	33	0
N.S.	1	1.00	0.75	0.68	0.85	1.12	0.86	0.85	0.39	0.00
time (sec)	N/A	0.045	0.020	0.003	1.587	1.010	0.154	0.388	1.562	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	140	135	109	0	211	190	108	95	0
N.S.	1	1.00	0.96	0.78	0.00	1.51	1.36	0.77	0.68	0.00
time (sec)	N/A	0.095	0.174	0.018	0.000	1.461	0.702	0.420	0.144	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	258	27	0	991	19	247	311	0
N.S.	1	1.00	0.74	0.08	0.00	2.86	0.05	0.71	0.90	0.00
time (sec)	N/A	0.247	0.188	0.008	0.000	1.342	2.784	0.875	2.284	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	55	42	0	377	20	245	145	0
N.S.	1	1.00	0.17	0.13	0.00	1.14	0.06	0.74	0.44	0.00
time (sec)	N/A	0.235	0.016	0.013	0.000	1.321	3.100	0.498	0.225	0.001

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	31	42	27	43	26	29	21	0
N.S.	1	1.00	1.15	1.56	1.00	1.59	0.96	1.07	0.78	0.00
time (sec)	N/A	0.008	0.013	0.012	1.326	1.490	0.147	0.518	0.047	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	131	96	0	247	49	147	269	0
N.S.	1	1.00	1.00	0.73	0.00	1.89	0.37	1.12	2.05	0.00
time (sec)	N/A	0.086	0.077	0.040	0.000	1.570	1.189	0.960	0.200	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	53	40	0	331	24	0	399	0
N.S.	1	1.00	0.34	0.25	0.00	2.11	0.15	0.00	2.54	0.00
time (sec)	N/A	0.087	0.013	0.013	0.000	1.260	0.192	0.000	1.721	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	55	42	0	574	24	0	483	0
N.S.	1	1.00	0.32	0.25	0.00	3.36	0.14	0.00	2.82	0.00
time (sec)	N/A	0.151	0.013	0.013	0.000	1.651	0.195	0.000	1.758	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	111	78	0	181	49	123	233	0
N.S.	1	1.00	0.95	0.67	0.00	1.55	0.42	1.05	1.99	0.00
time (sec)	N/A	0.057	0.054	0.062	0.000	1.233	1.157	0.907	0.190	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	511	511	57	44	0	1443	76	0	5341	0
N.S.	1	1.00	0.11	0.09	0.00	2.82	0.15	0.00	10.45	0.00
time (sec)	N/A	0.359	0.025	0.003	0.000	1.354	3.632	0.000	3.743	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	57	44	0	894	26	223	447	0
N.S.	1	1.00	0.14	0.11	0.00	2.18	0.06	0.54	1.09	0.00
time (sec)	N/A	0.321	0.015	0.012	0.000	1.583	1.455	0.685	1.677	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	90	68	82	126	82	82	44	0
N.S.	1	1.00	0.93	0.70	0.85	1.30	0.85	0.85	0.45	0.00
time (sec)	N/A	0.052	0.065	0.007	1.557	1.230	0.176	0.300	1.616	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	140	129	109	0	137	148	108	109	0
N.S.	1	1.00	0.92	0.78	0.00	0.98	1.06	0.77	0.78	0.00
time (sec)	N/A	0.099	0.173	0.013	0.000	1.161	0.621	0.371	0.185	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	257	29	0	991	20	247	312	0
N.S.	1	1.00	0.74	0.08	0.00	2.86	0.06	0.71	0.90	0.00
time (sec)	N/A	0.270	0.164	0.008	0.000	1.556	2.746	0.719	1.956	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	355	355	57	44	0	715	26	253	208	0
N.S.	1	1.00	0.16	0.12	0.00	2.01	0.07	0.71	0.59	0.00
time (sec)	N/A	0.277	0.016	0.009	0.000	1.627	3.103	0.462	1.666	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	25	10	17	17	17	19	9	0
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69	0.00
time (sec)	N/A	0.005	0.005	0.001	1.596	1.420	0.130	0.449	0.025	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	129	110	0	255	51	147	269	0
N.S.	1	1.00	1.00	0.85	0.00	1.98	0.40	1.14	2.09	0.00
time (sec)	N/A	0.118	0.077	0.026	0.000	1.442	1.172	0.746	1.709	0.001

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	55	42	0	302	26	0	399	0
N.S.	1	1.00	0.33	0.25	0.00	1.83	0.16	0.00	2.42	0.00
time (sec)	N/A	0.104	0.013	0.010	0.000	1.579	0.198	0.000	0.181	0.001

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	57	44	0	546	26	0	483	0
N.S.	1	1.00	0.34	0.26	0.00	3.23	0.15	0.00	2.86	0.00
time (sec)	N/A	0.142	0.014	0.012	0.000	1.809	0.194	0.000	1.787	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	114	90	0	199	51	135	245	0
N.S.	1	1.00	0.91	0.72	0.00	1.59	0.41	1.08	1.96	0.00
time (sec)	N/A	0.067	0.054	0.031	0.000	1.403	1.165	0.633	0.199	0.001

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	71	47	0	104	163	107	133	0
N.S.	1	1.00	0.53	0.35	0.00	0.77	1.21	0.79	0.99	0.00
time (sec)	N/A	0.124	0.034	0.056	0.000	0.774	0.904	0.494	2.235	0.001

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	72	62	0	111	0	123	1	0
N.S.	1	1.00	0.44	0.38	0.00	0.68	0.00	0.75	0.01	0.00
time (sec)	N/A	0.095	0.038	0.045	0.000	1.229	0.000	0.432	2.190	0.001

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	89	62	0	141	0	131	1	0
N.S.	1	1.00	0.49	0.34	0.00	0.78	0.00	0.73	0.01	0.00
time (sec)	N/A	0.122	0.046	0.014	0.000	1.552	0.000	0.448	2.230	0.001

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	43	42	108	112	43	39	0
N.S.	1	1.00	1.00	0.88	0.86	2.20	2.29	0.88	0.80	0.00
time (sec)	N/A	0.030	0.024	0.006	1.619	0.749	0.283	0.268	1.594	0.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	161	0	291	423	85	127	0
N.S.	1	1.00	1.00	1.87	0.00	3.38	4.92	0.99	1.48	0.00
time (sec)	N/A	0.081	0.090	0.003	0.000	1.200	1.372	0.324	1.772	0.002

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	293	266	240	754	109	247	555	0
N.S.	1	1.00	1.16	1.05	0.95	2.98	0.43	0.98	2.19	0.00
time (sec)	N/A	0.211	0.096	0.006	1.298	1.356	0.704	0.352	0.313	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	251	560	0	2540	0	3183	6366	0
N.S.	1	1.00	1.21	2.69	0.00	12.21	0.00	15.30	30.61	0.00
time (sec)	N/A	0.543	0.173	0.027	0.000	1.672	0.000	3.756	2.854	0.002

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	311	346	334	295	3169	167	295	1308	0
N.S.	1	1.00	1.11	1.07	0.95	10.19	0.54	0.95	4.21	0.00
time (sec)	N/A	0.290	0.112	0.084	1.525	2.134	2.981	0.534	3.100	0.001

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	716	716	88	67	0	0	0	0	11453	0
N.S.	1	1.00	0.12	0.09	0.00	0.00	0.00	0.00	16.00	0.00
time (sec)	N/A	1.634	0.054	0.016	0.000	0.000	0.000	0.000	29.420	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	753	753	551	45	0	3378	0	647	2520	0
N.S.	1	1.00	0.73	0.06	0.00	4.49	0.00	0.86	3.35	0.00
time (sec)	N/A	1.436	0.903	0.004	0.000	2.072	0.000	0.808	1.220	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	433	433	88	67	0	0	0	0	50213	0
N.S.	1	1.00	0.20	0.15	0.00	0.00	0.00	0.00	115.97	0.00
time (sec)	N/A	0.989	0.075	0.007	0.000	0.000	0.000	0.000	9.242	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	57	66	82	137	656	207	59	0
N.S.	1	1.00	0.92	1.06	1.32	2.21	10.58	3.34	0.95	0.00
time (sec)	N/A	0.039	0.152	0.013	0.550	0.938	1.320	0.351	1.662	0.068

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	123	138	208	495	3128	828	131	0
N.S.	1	1.00	0.93	1.05	1.58	3.75	23.70	6.27	0.99	0.00
time (sec)	N/A	0.102	0.248	0.015	0.695	0.766	10.968	0.453	1.711	0.663

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	205	226	386	1209	9190	2134	227	0
N.S.	1	1.00	0.94	1.04	1.77	5.55	42.16	9.79	1.04	0.00
time (sec)	N/A	0.201	0.426	0.020	0.882	0.876	89.545	0.779	1.850	3.400

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [38] had the largest ratio of [.5882]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	8	1.00	17	0.471
2	A	13	7	1.00	18	0.389
3	A	19	6	1.00	17	0.353
4	A	13	10	1.00	18	0.556
5	A	19	6	1.00	26	0.231
6	A	19	6	1.00	26	0.231
7	A	7	4	1.00	27	0.148
8	A	19	6	1.00	27	0.222
9	A	19	6	1.00	18	0.333
10	A	19	7	1.00	18	0.389
11	A	10	7	1.00	18	0.389
12	A	19	6	1.00	16	0.375
13	A	19	6	1.00	13	0.462
14	A	19	6	1.00	18	0.333
15	A	5	5	1.00	18	0.278
16	A	7	4	1.00	18	0.222
17	A	7	4	1.00	18	0.222
18	A	7	4	1.00	18	0.222
19	A	7	4	1.00	18	0.222
20	A	19	6	1.00	20	0.300
21	A	19	7	1.00	20	0.350
22	A	11	8	1.00	20	0.400
23	A	19	6	1.00	18	0.333
24	A	19	6	1.00	15	0.400
25	A	19	6	1.00	20	0.300
26	A	5	5	1.00	20	0.250
27	A	7	4	1.00	20	0.200
28	A	7	4	1.00	20	0.200
29	A	7	4	1.00	20	0.200
30	A	7	4	1.00	20	0.200
31	A	9	6	1.00	25	0.240
32	A	9	6	1.00	26	0.231
33	A	9	6	1.00	33	0.182
34	A	5	5	1.00	17	0.294
35	A	6	6	1.00	22	0.273
36	A	11	8	1.00	17	0.471
37	A	5	4	1.00	22	0.182
38	A	14	10	1.00	17	0.588
39	A	15	9	1.00	22	0.409

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	21	8	1.00	17	0.471
41	A	9	6	1.00	22	0.273
42	A	2	1	1.00	22	0.045
43	A	2	1	1.00	24	0.042
44	A	2	1	1.00	24	0.042

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{d+ex^3}{a+cx^6} dx$

Optimal. Leaf size=305

$$-\frac{\left(\sqrt{3}\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}\,e+\sqrt{3}\sqrt{c}\,d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}-\frac{(\sqrt{}}{}$$

Rubi [A] time = 0.25, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1416, 635, 203, 260, 634, 617, 204, 628}

$$-\frac{\left(\sqrt{3}\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}\,e+\sqrt{3}\sqrt{c}\,d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{3}\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{c}\,d-\sqrt{3}\sqrt{a}\,e\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)}{6a^{5/6}c^{2/3}}+\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d + Sqrt[3]
*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(2/3)) +
((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(
6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - (
(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c
^(1/3)*x^2])/(12*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1416

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{\int \frac{\frac{2\sqrt[3]{c}d}{\sqrt[3]{a}}-\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}\sqrt[3]{c}} + \frac{\int \frac{\frac{2\sqrt[3]{c}d}{\sqrt[3]{a}}+\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{1+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}\sqrt[3]{c}} + \frac{\int \frac{\frac{\sqrt[3]{c}d}{\sqrt[3]{a}}-ex}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}}$$

$$= \frac{d\int \frac{1}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a} - \frac{e\int \frac{x}{1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{-\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}x}{\sqrt[3]{a}}}{1-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}\,dx}{12a^{5/6}c^{2/3}} + \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\int \frac{\ldots}{1+\ldots}}{12a^{5/6}c^{2/3}\ldots}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}} + \ldots$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} + \frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} \ldots$$

Mathematica [A] time = 0.10, size = 334, normalized size = 1.10

$$-\frac{\left(\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}-\frac{\left(-a^{2/3}e-\sqrt{3}\sqrt[6]{a}\sqrt{c}d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}+\frac{\left(\sqrt{3}a^{2/3}e+\sqrt[6]{a}\sqrt{c}d\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x-\sqrt{3}\sqrt[6]{a}}{\sqrt[6]{a}}\right)}{6ac^{2/3}}+\frac{\left(\sqrt[6]{a}\sqrt{c}d-\sqrt{3}a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}}+\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) + ((a^(1/6)*Sqrt[c]*d +
 Sqrt[3]*a^(2/3)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(6*
a*c^(2/3)) + ((a^(1/6)*Sqrt[c]*d - Sqrt[3]*a^(2/3)*e)*ArcTan[(Sqrt[3]*a^(1/
6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/
(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(1/3)
- Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(2/3)) - ((-(Sqrt[3]*a^
(1/6)*Sqrt[c]*d) - a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(
1/3)*x^2])/(12*a*c^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(a + c*x^6), x]
```

fricas [B] time = 1.62, size = 3224, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*
c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*arctan(1/3*(2*(sqrt(3)*(a^4*c^4
*d^2 - a^5*c^3*e^2)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^
3)) - 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a^3*c^2*d^2*e^3))*sqrt(((c^3*d^7 - a*c^2
*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 + (2*a^5*c^3*d*e*sqrt(-(c^2*d
^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a^2*c^3*d^5 - 4*a^3*c^2*d^
3*e^2 + 3*a^4*c*d*e^4)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2
*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - ((a^4*c^3*d^2*e +
a^5*c^2*e^3)*x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) +
 (a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*((a^2*c^2*sqrt(-(c^2*
d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c
^2))^(1/3))/(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6))*((a^
2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*
e - a*e^3)/(a^2*c^2))^(2/3) - 2*(sqrt(3)*(a^4*c^4*d^2 - a^5*c^3*e^2)*x*sqrt
(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 2*sqrt(3)*(a^2*c^3
*d^4*e - 3*a^3*c^2*d^2*e^3)*x)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9
*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - sqrt(3)*(c
^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6))/(c^3*d^7 - a*c^2*d
^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)) - 1/3*sqrt(3)*(-(a^2*c^2*sqrt(-(c^
2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2
*c^2))^(1/3)*arctan(1/3*(2*(sqrt(3)*(a^4*c^4*d^2 - a^5*c^3*e^2)*sqrt(-(c^2*
d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^3*d^4*e
- 3*a^3*c^2*d^2*e^3))*sqrt(((c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*
a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) - a^2*c^3*d^5 + 4*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*(-(a^2*c
^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e +
 a*e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e + a^5*c^2*e^3)*x*sqrt(-(c^2*d^6
- 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a*c^3*d^6 - 2*a^2*c^2*d^4*e^
2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d
^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3))/(c^3*d^7 - a*c^2*
d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6))*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c
*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3)
- 2*(sqrt(3)*(a^4*c^4*d^2 - a^5*c^3*e^2)*x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a^3*c^2*d^2*e^3)*
x)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) -
3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) + sqrt(3)*(c^3*d^7 - a*c^2*d^5*e^2 - 5*
a^2*c*d^3*e^4 - 3*a^3*d*e^6))/(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 -
3*a^3*d*e^6)) - 1/12*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^3*d^7 - a*c^2*
d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e*sqrt(-(c^2*d^
6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a^2*c^3*d^5 - 4*a^3*c^2*d^3
*e^2 + 3*a^4*c*d*e^4)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e + a
^5*c^2*e^3)*x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) +
(a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*((a^2*c^2*sqrt(-(c^2*d
^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^
2))^(1/3)) - 1/12*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4
)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^3*d^7 - a*c^2*d^
```

```
5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 + (2*a^5*c^3*d*e*sqrt(-(c^2*d^6
- 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*c^3*d^5 + 4*a^3*c^2*d^3*e
^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) - ((a^4*c^3*d^2*e + a^
5*c^2*e^3)*x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (
a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt(-(c^2*d
^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^
2))^(1/3)) + 1/6*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/
(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a*c*d^3*
e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^
2*e^4)/(a^5*c^3)) + a*c^2*d^4 - 3*a^2*c*d^2*e^2)*((a^2*c^2*sqrt(-(c^2*d^6 -
 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^
(1/3)) + 1/6*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^
5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a*c*d^3*e^2
 - 3*a^2*d*e^4)*x + (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^5*c^3)) - a*c^2*d^4 + 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6
*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1
/3))
```

giac [A] time = 0.43, size = 288, normalized size = 0.94

$$-\frac{|c|e\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(ac^5\right)^{\frac{1}{3}}}+\frac{\left(ac^5\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3ac}+\frac{\left(\left(ac^5\right)^{\frac{1}{6}}c^3d-\sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4}+\frac{\left(\left(ac^5\right)^{\frac{1}{6}}c^3d+\sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^4}+\frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}c^3d+\left(ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^4}-\frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}c^3d-\left(ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

```
[Out] -1/6*abs(c)*e*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + 1/3*(a*c^5)^(1/6)*d*ar
ctan(x/(a/c)^(1/6))/(a*c) + 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2/3
)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c^5)
^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/
(a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/3)*e)
*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(3)*(a*
c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)
^(1/3))/(a*c^4)
```

maple [A] time = 0.12, size = 329, normalized size = 1.08

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^2+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{6}}\sqrt{3}\,cd\ln\left(x^2+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(c*x^6+a),x)
```

```
[Out] 1/12*c*(a/c)^(7/6)/a^2*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*d+
1/12*(a/c)^(2/3)/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*e+1/6*(a/c)^(1
/6)/a*arctan(2*x/(a/c)^(1/6)+3^(1/2))*d-1/6*(a/c)^(2/3)/a*arctan(2*x/(a/c)^
(1/6)+3^(1/2))*3^(1/2)*e+1/12/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(
a/c)^(2/3)*e-1/12/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)
^(1/6)*d+1/6/a*(a/c)^(2/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*e+1/6/a*
(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*d-1/6*(a/c)^(2/3)/a*e*ln(x^2+(a
/c)^(1/3))+1/3*(a/c)^(1/6)/a*d*arctan(x/(a/c)^(1/6))
```

maxima [A] time = 1.49, size = 282, normalized size = 0.92

$$-\frac{e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}c^{\frac{2}{3}}}+\frac{d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{3a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}+\frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}\,d+a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12ac^{\frac{2}{3}}}-\frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}\,d-a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12ac^{\frac{2}{3}}}-\frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e-a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2c^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}+\frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e+a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2c^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/6*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) + 1/3*d*arctan(c^(1/3)*
x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/12*(sqrt(3)*a^
(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x +
a^(1/3))/(a*c^(2/3)) - 1/12*(sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log(c^(
1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - 1/6*(sqrt(3)*
a^(5/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6
)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/6*(
sqrt(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3
)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3)))
```

mupad [B] time = 1.54, size = 1331, normalized size = 4.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a + c*x^6),x)
```

```
[Out] log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a
*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(-a^5*c^5)^(1/2) + a^2*c^3*
d*x)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3
*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4)
)^(1/3) - e*x*(-a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(-a^5*
c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(
1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*
e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(-a^5*c^5)^(1/2) +
 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e
- 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2
)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e -
3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(-a^5*c^5)^(1/2)
 - (a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a
*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*
e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2)
)/(a^5*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e
^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))
/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2)
 - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(
-a^5*c^5)^(1/2) - 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) -
 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*
c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3
*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log(a^3
*c^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(-a^5*c^5)^(1/2) + 3^(1/2)*a^3*c
^3*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-
a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)
*(-(a^4*c^2*e^3 - c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^
5*c^5)^(1/2))/(216*a^5*c^4))^(1/3)
```

sympy [A] time = 3.11, size = 165, normalized size = 0.54

$$\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\left(432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4a^4c^2e - 6ta^3e^4 + 36ta^2cd^2e^2 - 6tac^2d^4}{3a^2de^4 + 2acd^3e^2 - c^2d^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(c*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a**5*c**4 + _t**3*(432*a**4*c**2*e**3 - 1296*a**3*c**3*
d**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6,
Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e - 6*_t*a**3*e**4 + 36*_t*a**
2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c**2*d
**5))))
```

3.2 $\int \frac{d+ex^3}{a-cx^6} dx$

Optimal. Leaf size=323

$$\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}c^{2/3}}-\frac{(}{}$$

Rubi [A] time = 0.19, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1417, 200, 31, 634, 617, 204, 628}

$$\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}c^{2/3}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(-\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12a^{5/6}\sqrt[6]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{6a^{5/6}\sqrt[6]{c}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}\sqrt[6]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] -((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)
)])/(2*Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(a^(1/6)
+ 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]*a^(5/6)*c^(2/3)) - ((Sqrt[c]*
d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqrt[
a]*e)/Sqrt[c])*Log[a^(1/6) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqrt[
a]*e)/Sqrt[c])*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*
c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/
3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1417

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-(a/c), 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d
 - e*q)/2, Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Rubi steps

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{1}{2}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a+\sqrt{a}\,\sqrt{c}\,x^3}\,dx + \frac{1}{2}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a-\sqrt{a}\,\sqrt{c}\,x^3}\,dx$$

$$= \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[6]{a}\,\sqrt[6]{c}\,x}\,dx}{6a^{2/3}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{2\sqrt[3]{a}-\sqrt[6]{a}\,\sqrt[6]{c}\,x}{a^{2/3}-\sqrt{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}\,\sqrt[3]{c}\,x^2}\,dx}{6a^{2/3}} + \frac{\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\,\sqrt[6]{c}\,x}\,d\cdots}{6a^{2/3}}$$

$$= -\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{6a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\int \frac{\sqrt{a}\,\sqrt[6]{c}+2\cdots}{a^{2/3}+\sqrt{a}\,\sqrt[6]{c}\,x\cdots}}{12a^{5/6}c^{2/3}}$$

$$= -\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{6a^{5/6}\sqrt[6]{c}} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\,\sqrt[6]{\cdots}\right.}{12a^{5/6}\sqrt[6]{c}}$$

$$= -\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt{3}\,\sqrt[6]{a}}\right)}{2\sqrt{3}\,a^{5/6}c^{2/3}} - \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt[6]{a}-\cdots\right.}{6a^{5/6}c^{2/3}}$$

Mathematica [A] time = 0.12, size = 337, normalized size = 1.04

$$\frac{-2\sqrt{3}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)-\sqrt{c}\,d\log\left(-\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)+\sqrt{c}\,d\log\left(\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)-2\sqrt{c}\,d\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)+2\sqrt{c}\,d\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)+\sqrt{a}\,e\log\left(-\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)+\sqrt{a}\,e\log\left(\sqrt[6]{a}\,\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)-2\sqrt{a}\,e\log\left(\sqrt[6]{a}-\sqrt[6]{c}\,x\right)-2\sqrt{a}\,e\log\left(\sqrt[6]{a}+\sqrt[6]{c}\,x\right)}{12a^{5/6}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] (-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt
[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))
/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/6)
- c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/
6) + c^(1/6)*x] - Sqrt[c]*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
+ Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*d*Log[
a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) + a^(1/6
)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(a - c*x^6), x]
```

fricas [B] time = 1.94, size = 3178, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*
c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*arctan(1/3*(2*(sqrt(3)*(a^4*c^4
*d^2 + a^5*c^3*e^2)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3
)) - 2*sqrt(3)*(a^2*c^3*d^4*e + 3*a^3*c^2*d^2*e^3))*sqrt(((c^3*d^7 + a*c^2*
d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e*sqrt((c^2*d^6
 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*c^3*d^5 - 4*a^3*c^2*d^3*
e^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e - a^
5*c^2*e^3)*x*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a
*c^3*d^6 + 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt((c^2*d^6
 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2)
)^(1/3))/(c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6))*(-(a^2*
c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e +
 a*e^3)/(a^2*c^2))^(2/3) - 2*(sqrt(3)*(a^4*c^4*d^2 + a^5*c^3*e^2)*x*sqrt((c
^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 2*sqrt(3)*(a^2*c^3*d^4
*e + 3*a^3*c^2*d^2*e^3)*x)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2
*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) - sqrt(3)*(c^3*d
^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6))/(c^3*d^7 + a*c^2*d^5*e
^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)) - 1/3*sqrt(3)*((a^2*c^2*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))
^(1/3)*arctan(1/3*(2*(sqrt(3)*(a^4*c^4*d^2 + a^5*c^3*e^2)*sqrt((c^2*d^6 + 6
*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 2*sqrt(3)*(a^2*c^3*d^4*e + 3*a^3
*c^2*d^2*e^3))*sqrt(((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e
^6)*x^2 + (2*a^5*c^3*d*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^
5*c^3)) + a^2*c^3*d^5 + 4*a^3*c^2*d^3*e^2 + 3*a^4*c*d*e^4)*((a^2*c^2*sqrt((
c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a
^2*c^2))^(2/3) - ((a^4*c^3*d^2*e - a^5*c^2*e^3)*x*sqrt((c^2*d^6 + 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^2 - 3*a^3*c
*d^2*e^4)*x)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*
c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))/(c^3*d^7 + a*c^2*d^5*e^2 - 5*a
^2*c*d^3*e^4 + 3*a^3*d*e^6))*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^
2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - 2*(sqrt(3)*(a
^4*c^4*d^2 + a^5*c^3*e^2)*x*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/
(a^5*c^3)) + 2*sqrt(3)*(a^2*c^3*d^4*e + 3*a^3*c^2*d^2*e^3)*x)*((a^2*c^2*sqr
t((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)
/(a^2*c^2))^(2/3) + sqrt(3)*(c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*
a^3*d*e^6))/(c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)) - 1/
12*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3
*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d
^3*e^4 + 3*a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 +
9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*c^3*d^5 - 4*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4
)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*
c*d^2*e + a*e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e - a^5*c^2*e^3)*x*sqrt((
c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a*c^3*d^6 + 2*a^2*c^
2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 +
9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) - 1/12*((a
^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*
e - a*e^3)/(a^2*c^2))^(1/3)*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4
+ 3*a^3*d*e^6)*x^2 + (2*a^5*c^3*d*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d
^2*e^4)/(a^5*c^3)) + a^2*c^3*d^5 + 4*a^3*c^2*d^3*e^2 + 3*a^4*c*d*e^4)*((a^2
*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e
- a*e^3)/(a^2*c^2))^(2/3) - ((a^4*c^3*d^2*e - a^5*c^2*e^3)*x*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^
```

```
2 - 3*a^3*c*d^2*e^4)*x)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2
*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)) + 1/6*(-(a^2*c^2*sq
rt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3
)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + (a^4*c^
2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a*c^2*d^4 -
 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)
/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) + 1/6*((a^2*c^2*sqrt((c^
2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2
*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*sq
rt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^4 + 3*a^2
*c*d^2*e^2)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c
^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))
```

giac [**A**]   time = 0.38, size = 308, normalized size = 0.95

$$\frac{|c|e\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}}+\frac{\left(-ac^5\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac}+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}c^3d-\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}c^3d+\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}+\frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}c^3d+\left(-ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2+\sqrt{3}\,x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}-\frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}c^3d-\left(-ac^5\right)^{\frac{2}{3}}e\right)\log\left(x^2-\sqrt{3}\,x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

```
[Out] 1/6*abs(c)*e*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) + 1/3*(-a*c^5)^(1/6)*d*
arctan(x/(-a/c)^(1/6))/(a*c) + 1/6*((-a*c^5)^(1/6)*c^3*d - sqrt(3)*(-a*c^5)
^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/6*(
(-a*c^5)^(1/6)*c^3*d + sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(-a/
c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d + (-a*
c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4) - 1/
12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(-
a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

maple [**A**]   time = 0.11, size = 386, normalized size = 1.20

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,d\arctan\left(\frac{2\sqrt{3}\,x}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}}{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,d\arctan\left(\frac{2\sqrt{3}\,x}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\ln\left(-x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(\frac{2\sqrt{3}\,x}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}}{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(\frac{2\sqrt{3}\,x}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(-x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}-\frac{d\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)}{6\left(\frac{a}{c}\right)^{\frac{5}{6}}c}+\frac{d\ln\left(x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)}{6\left(\frac{a}{c}\right)^{\frac{5}{6}}c}-\frac{e\ln\left(-x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)}{6\left(\frac{a}{c}\right)^{\frac{1}{3}}c}-\frac{e\ln\left(x+\left(\frac{a}{c}\right)^{\frac{1}{6}}\right)}{6\left(\frac{a}{c}\right)^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(-c*x^6+a),x)
```

```
[Out] -1/6/c/(a/c)^(1/3)*ln(x+(a/c)^(1/6))*e+1/6/c/(a/c)^(5/6)*ln(x+(a/c)^(1/6))*
d+1/12*(a/c)^(2/3)/a*ln((a/c)^(1/6)*x-x^2-(a/c)^(1/3))*e-1/12*(a/c)^(1/6)/a
*ln((a/c)^(1/6)*x-x^2-(a/c)^(1/3))*d-1/6*(a/c)^(2/3)/a*3^(1/2)*e*arctan(-1/
3*3^(1/2)+2/3*x*3^(1/2)/(a/c)^(1/6))+1/6*(a/c)^(1/6)/a*3^(1/2)*d*arctan(-1/
3*3^(1/2)+2/3*x*3^(1/2)/(a/c)^(1/6))-1/6/c/(a/c)^(1/3)*ln(-x+(a/c)^(1/6))*e
-1/6/c/(a/c)^(5/6)*ln(-x+(a/c)^(1/6))*d+1/12/a*(a/c)^(2/3)*e*ln(x^2+(a/c)^(
1/6)*x+(a/c)^(1/3))+1/6/a*(a/c)^(2/3)*e*3^(1/2)*arctan(2/3*x*3^(1/2)/(a/c)^
(1/6)+1/3*3^(1/2))+1/12/a*d*(a/c)^(1/6)*ln(x^2+(a/c)^(1/6)*x+(a/c)^(1/3))+1
/6/a*d*(a/c)^(1/6)*3^(1/2)*arctan(2/3*x*3^(1/2)/(a/c)^(1/6)+1/3*3^(1/2))
```

maxima [**A**]   time = 1.34, size = 313, normalized size = 0.97

$$\frac{\sqrt{3}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{a}\,c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\sqrt{a}\,c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{a}\,c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(x^2-x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\sqrt{a}\,c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{a}\,c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}-\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\sqrt{a}\,c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) +
1/6*sqrt(3)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/sqrt
```

```
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) +
1/12*(sqrt(c)*d + sqrt(a)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)
/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/12*(sqrt(c)*d - sq
rt(a)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sq
rt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 1/6*(sqrt(c)*d - sqrt(a)*e)*log(x + (sqr
t(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/6*(sqrt(c)*d +
 sqrt(a)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(
2/3))
```

mupad [B] time = 2.97, size = 1293, normalized size = 4.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a - c*x^6),x)
```

```
[Out] log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*
d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x
)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5
*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5
*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)
 - e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2
) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log
(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e
^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(a^5*c^5)^(1/2) + 3^(1/2)*a^3*
c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a
^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*
(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c
^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(a^5*c^5)^(1/2) - (a^3*c^3*(-(a^4
*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1
/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i)
/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(
1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) +
log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*
d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(a^5*c^5)^(1/2) - 3^(1/2)*a
^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2
*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/
2)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^
5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^
5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3
) + 2*e*x*(a^5*c^5)^(1/2) + 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i
- 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1
/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3)
```

sympy [A] time = 3.12, size = 168, normalized size = 0.52

$$-\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\left(-432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 - 3a^2cd^2e^4 + 3ac^2d^4e^2 - c^3d^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4a^4c^2e + 6ta^3e^4 + 36ta^2cd^2e^2 + 6tac^2d^4}{3a^2de^4 - 2acd^3e^2 - c^2d^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(-c*x**6+a),x)
```

```
[Out] -RootSum(46656*_t**6*a**5*c**4 + _t**3*(-432*a**4*c**2*e**3 - 1296*a**3*c**
3*d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6
, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e + 6*_t*a**3*e**4 + 36*_t*a
**2*c*d**2*e**2 + 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 - 2*a*c*d**3*e**2 - c**2
*d**5))))
```

3.3 $\int \frac{d+ex^4}{a+cx^8}\,dx$

Optimal. Leaf size=754

$$\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{7/8}c^{5/8}}-\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\cdots\right.}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{7/8}c^{5/8}}$$

Rubi [A] time = 1.25, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1415, 1169, 634, 618, 204, 628}

$$\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{7/8}c^{5/8}}-\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}\,a^{7/8}c^{5/8}}-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}\,a^{7/8}c^{5/8}}-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}-2\sqrt[8]{c}\,x}{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\left(-\frac{\sqrt{a}\,e}{\sqrt{c}}+\sqrt{2}\,d+d\right)\log\left(\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}\,a^{7/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[2 -
Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a^(7/8)*c^
(5/8)) + (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(S
qrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^
(7/8)*c^(5/8)) + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*A
rcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))
])/(8*a^(7/8)*c^(5/8)) - (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d - Sqrt
[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*
a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log
[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2
 - Sqrt[2])]*a^(7/8)*c^(5/8)) - (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log[
a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2
- Sqrt[2])]*a^(7/8)*c^(5/8)) - (((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log[a
^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 +
 Sqrt[2])]*a^(7/8)*c^(5/8)) + ((d + Sqrt[2]*d - (Sqrt[a]*e)/Sqrt[c])*Log[a^
(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 +
Sqrt[2])]*a^(7/8)*c^(1/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1415

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x
^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3),
 Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &
& NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

Rubi steps

$$\int \frac{d+ex^4}{a+cx^8}\,dx = \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}+\left(-d+\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}+\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2(2-\sqrt{2})}a^{3/8}d}{c^{3/8}}-\left(\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}} + \frac{\sqrt[8]{c}\int \frac{\frac{\sqrt{2(2-\sqrt{2})}a^{3/8}d}{c^{3/8}}+\left(\frac{\sqrt{2}\sqrt[4]{a}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}} + \frac{\sqrt[8]{c}\int \cdots}{\cdots}$$

$$= -\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}a^{3/4}c^{3/4}} \cdots$$

$$= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\cdots\right)}{8\sqrt{2\left(2-\cdots\right)}\cdots}$$

$$= -\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}} + \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\cdots}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\cdots}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} \cdots$$

Mathematica [A] time = 0.63, size = 534, normalized size = 0.71

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + c*x^8),x]
```

[Out] (-2*a^(1/8)*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + 2*a^(1/8)*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]))/(8*a*c^(5/8))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^4}{a + cx^8} \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^4)/(a + c*x^8),x]

[Out] IntegrateAlgebraic[(d + e*x^4)/(a + c*x^8), x]

fricas [B] time = 2.41, size = 3406, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="fricas")

[Out] -1/2*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*arctan(-((3*a^3*c^5*d^6*e - 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6*c^2*e^7 + (a^6*c^6*d^3 - 3*a^7*c^5*d*e^2)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt(((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x^2 - (2*a^6*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a^2*c^4*d^6 + 7*a^3*c^3*d^4*e^2 - 7*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)) - ((a^6*c^6*d^3 - 3*a^7*c^5*d*e^2)*x*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + (3*a^3*c^5*d^6*e - 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6*c^2*e^7)*x)*sqrt((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)/(c^5*d^10 - 3*a*c^4*d^8*e^2 - 14*a^2*c^3*d^6*e^4 - 14*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 + a^5*e^10)) + 1/2*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*arctan(((3*a^3*c^5*d^6*e - 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6*c^2*e^7 - (a^6*c^6*d^3 - 3*a^7*c^5*d*e^2)*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt(((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x^2 + (2*a^6*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a^2*c^4*d^6 - 7*a^3*c^3*d^4*e^2 + 7*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d

```
^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8
- 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8))*(-(a^3
*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*
e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4) + ((a^6
*c^6*d^3 - 3*a^7*c^5*d*e^2)*x*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - (3*a^3*c^5*d^6*e - 19*
a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 - a^6*c^2*e^7)*x)*(-(a^3*c^2*sqrt(-(c^4
*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4))/(c^5*d^10 - 3*a*c^4*d
^8*e^2 - 14*a^2*c^3*d^6*e^4 - 14*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 + a^5*e^
10)) + 1/8*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^
2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x + (
a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c
*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^
4)*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4
)) - 1/8*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
- 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2)
)^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x - (a^
5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)
*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3
*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4))
 - 1/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 1
2*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(
1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x + (a^5*c
^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*
e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((
a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) + 1
/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)
*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x - (a^5*c^3*e
*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6
+ a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*
c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e
^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4))
```

giac [A] time = 0.74, size = 601, normalized size = 0.80

$$-\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}-\frac{\left(\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*a
rctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)
))/a - 1/8*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c)^(1
/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)
^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a
/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)
*(a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) +
 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2
) + 2)*(a/c)^(1/8)))/a - 1/16*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sq
rt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(
1/4))/a + 1/16*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c
)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a + 1/16*
(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^
2 + x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a - 1/16*(sqrt(sqrt(2)
```

```
+ 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(-sq
rt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a
```

maple [C] time = 0.02, size = 34, normalized size = 0.05

$$\frac{\left(\text{RootOf}\left(\_Z^8 c + a\right)^4 e + d\right) \ln\left(-\text{RootOf}\left(\_Z^8 c + a\right) + x\right)}{8c\,\text{RootOf}\left(\_Z^8 c + a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(c*x^8+a),x)
```

```
[Out] 1/8/c*sum((_R^4*e+d)/_R^7*ln(-_R+x),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4 + d}{cx^8 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + a), x)
```

mupad [B] time = 2.78, size = 2510, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a + c*x^8),x)
```

```
[Out] (atan((c^3*d^6*x - a^3*e^6*x + a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*x
*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a
^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^
5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*
((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) - 2*a^2*c
^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^
4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4
) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4
*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5
))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^
4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4
))/4 - (atan((a^3*e^6*x - c^3*d^6*x - a*c^2*d^4*e^2*x + a^2*c*d^2*e^4*x + (
2*d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^
3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*
d^5*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*
e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^
5*c^3*e*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*
d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4)
- 2*a^2*c^2*d^3*e^2*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2)
+ 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*
c^5))^(1/4) - 3*a^3*c*d*e^4*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5
)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2
))/(a^7*c^5))^(1/4)))*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2
) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^
7*c^5))^(1/4))/4 - atan((c^3*d^6*x*1i - a^3*e^6*x*1i + a*c^2*d^4*e^2*x*1i -
 a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)
```

```
)*2i)/(a^3*c^2))/(a*c^3*d^5*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)
^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2)
)/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c
^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1
/2))/(a^7*c^5))^(5/4) - 2*a^2*c^2*d^3*e^2*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*
d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-
a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4*((a^2*e^4*(-a^7*c^5)^(1/2)
 + c^2*d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2
*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^2*e^4*(-a^7*c^5)^(1/2) + c^2*
d^4*(-a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-
a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i + atan((a^3*e^6*x*1i - c^3*d^6*x*1
i - a*c^2*d^4*e^2*x*1i + a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^4*(-a^7*c^5)^(1
/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c*
d^2*e^2*(-a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(-a^7*c^5)^(
1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6*a*c
*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(-a^7*c^
5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 - 6
*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) - 2*a^2*c^2*d^3*e^2*(-(a^2*
e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4
*(-(a^2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e -
 4*a^5*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*(-(a^
2*e^4*(-a^7*c^5)^(1/2) + c^2*d^4*(-a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e - 4*a^5
*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(c*x**8+a),x)
```

```
[Out] Timed out
```

3.4 $\int \frac{d+ex^4}{a-cx^8}\,dx$

Optimal. Leaf size=329

$$\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\right.}{}$$

Rubi [A] time = 0.21, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1417, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{c}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}+1\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*c^(5/8)) -
 ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 - (Sqrt[2]*c^(1/8)*x)/a^(1/8)])/(4*Sqr
t[2]*a^(7/8)*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 + (Sqrt[2]*c^(1
/8)*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)*c^(1/8)) + ((Sqrt[c]*d + Sqrt[a]*e)*Arc
Tanh[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])
*Log[a^(1/4) - Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)
*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8
)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1417

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-(a/c), 2]}, Dist[(d + e*q)/2, Int[1/(a + c*q*x^n), x], x] + Dist[(d
 - e*q)/2, Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^4}{a-cx^8}\,dx &= \frac{1}{2}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a+\sqrt{a}\,\sqrt{c}\,x^4}\,dx+\frac{1}{2}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{a-\sqrt{a}\,\sqrt{c}\,x^4}\,dx\\
&= \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt[4]{a}-\sqrt[4]{c}\,x^2}{a+\sqrt{a}\,\sqrt{c}\,x^4}\,dx}{4\sqrt[4]{a}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt[4]{a}+\sqrt[4]{c}\,x^2}{a+\sqrt{a}\,\sqrt{c}\,x^4}\,dx}{4\sqrt[4]{a}}+\frac{\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt[4]{a}-\sqrt[4]{c}\,x^2}\,dx}{4a^{3/4}}+\frac{\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \cdots}{4a\cdots}\\
&= \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2}\,\sqrt[8]{a}\,x}{\sqrt[8]{c}}+x^2}\,dx}{8a^{3/4}\sqrt[4]{c}}+\frac{(d\cdots}{\cdots}\\
&= \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt[4]{a}-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\cdots\right.}{8\sqrt{2}\,a^{7/8}\sqrt[8]{c}}\cdots\\
&= \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{c}}+\frac{(\cdots}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 425, normalized size = 1.29

$$\frac{\left(a^{5/8}e-\sqrt[8]{a}\,\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,ac^{5/8}}-\frac{\left(a^{5/8}e-\sqrt[8]{a}\,\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{c}\,x+\sqrt[4]{a}+\sqrt[4]{c}\,x^2\right)}{8\sqrt{2}\,ac^{5/8}}-\frac{\left(a^{5/8}e+\sqrt[8]{a}\,\sqrt{c}\,d\right)\log\left(\sqrt[8]{a}-\sqrt[8]{c}\,x\right)}{8ac^{5/8}}-\frac{\left(-a^{5/8}e-\sqrt[8]{a}\,\sqrt{c}\,d\right)\log\left(\sqrt[8]{a}+\sqrt[8]{c}\,x\right)}{8ac^{5/8}}+\frac{\left(a^{5/8}e+\sqrt[8]{a}\,\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt[8]{c}\,x}{\sqrt[8]{a}}\right)}{4ac^{5/8}}-\frac{\left(a^{5/8}e-\sqrt[8]{a}\,\sqrt{c}\,d\right)\tan^{-1}\left(\frac{2\sqrt[8]{c}\,x-\sqrt{2}\,\sqrt[8]{a}}{\sqrt{2}\,\sqrt[8]{a}}\right)}{4\sqrt{2}\,ac^{5/8}}-\frac{\left(a^{5/8}e-\sqrt[8]{a}\,\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{a}+2\sqrt[8]{c}\,x}{\sqrt{2}\,\sqrt[8]{a}}\right)}{4\sqrt{2}\,ac^{5/8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a*c^(5/8))
 - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*ArcTan[(-(Sqrt[2]*a^(1/8)) + 2*c^(1/
8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) +
a^(5/8)*e)*ArcTan[(Sqrt[2]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sq
rt[2]*a*c^(5/8)) - ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*Log[a^(1/8) - c^(1/8)*x
])/(8*a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) - a^(5/8)*e)*Log[a^(1/8) + c^(1/8
)*x])/(8*a*c^(5/8)) + ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) - Sqr
t[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8)) - ((-(a^(1/8)*
Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x
^2])/(8*Sqrt[2]*a*c^(5/8))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^4}{a - cx^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(a - c*x^8), x]
```

fricas [B] time = 2.84, size = 3385, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/2*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)
*arctan(((3*a^3*c^5*d^6*e + 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 + a^6*c^
2*e^7 - (a^6*c^6*d^3 + 3*a^7*c^5*d*e^2)*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)))*sqrt(((c^4*d^8
 + 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*x^2 -
(2*a^6*c^4*d*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a^2*c^4*d^6 - 7*a^3*c^3*d^4*e^2 - 7*a^
4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e +
4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8 + 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 + a^4*e^8))*sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4
*a*d*e^3)/(a^3*c^2)) + ((a^6*c^6*d^3 + 3*a^7*c^5*d*e^2)*x*sqrt((c^4*d^8 + 1
2*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5
)) - (3*a^3*c^5*d^6*e + 19*a^4*c^4*d^4*e^3 + 9*a^5*c^3*d^2*e^5 + a^6*c^2*e^
7)*x)*sqrt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 +
 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))
)*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*
c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)/(
c^5*d^10 + 3*a*c^4*d^8*e^2 - 14*a^2*c^3*d^6*e^4 + 14*a^3*c^2*d^4*e^6 - 3*a^
4*c*d^2*e^8 - a^5*e^10)) - 1/2*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e -
 4*a*d*e^3)/(a^3*c^2))^(1/4)*arctan(-((3*a^3*c^5*d^6*e + 19*a^4*c^4*d^4*e^3
 + 9*a^5*c^3*d^2*e^5 + a^6*c^2*e^7 + (a^6*c^6*d^3 + 3*a^7*c^5*d*e^2)*sqrt((
c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)))*sqrt(((c^4*d^8 + 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 + 4*a^
3*c*d^2*e^6 + a^4*e^8)*x^2 + (2*a^6*c^4*d*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a^2*c^4*d
^6 + 7*a^3*c^3*d^4*e^2 + 7*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt
((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*
```

```
e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8 + 4*a*c^3*d^6
*e^2 - 10*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8))*(-(a^3*c^2*sqrt((c^
4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)
/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4) - ((a^6*c^6*d^3 + 3*a
^7*c^5*d*e^2)*x*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + (3*a^3*c^5*d^6*e + 19*a^4*c^4*d^4*e^3
 + 9*a^5*c^3*d^2*e^5 + a^6*c^2*e^7)*x)*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*
d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c
*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(3/4))/(c^5*d^10 + 3*a*c^4*d^8*e^2 - 14*a^2*
c^3*d^6*e^4 + 14*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)) + 1/8*((a^3
*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e
^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-(c^
3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x + (a^5*c^3*e*sqrt((c
^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8
)/(a^7*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt(
(c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e
^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/8*((a^3*c^2*s
qrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a
^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-(c^3*d^6
+ 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x - (a^5*c^3*e*sqrt((c^4*d^8
 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7
*c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt((c^4*d
^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a
^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/8*(-(a^3*c^2*sqrt((
c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a
*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x + (a^5*c^3*e*sqrt((c^4*d^8 + 12
*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)
) + a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt((c^4*d^8 +
 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c
^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)) + 1/8*(-(a^3*c^2*sqrt((c^4*
d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(
a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2
*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x - (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c
^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) +
a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt((c^4*d^8 + 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
 - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4))
```

giac [B] time = 0.75, size = 633, normalized size = 1.92

$$-\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e-d\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}-\frac{\left(\sqrt{\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{5}{8}}e+d\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(-\frac{a}{c}\right)^{\frac{1}{8}}+\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))
*arctan((2*x + sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(-a/c)^(
1/8)))/a - 1/8*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a
/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2
)*(-a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2
) + 2)*(-a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(-s
qrt(2) + 2)*(-a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sq
rt(-sqrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(-a/c)^(1/8)
)/(sqrt(-sqrt(2) + 2)*(-a/c)^(1/8)))/a - 1/16*(sqrt(-sqrt(2) + 2)*(-a/c)^(5
/8)*e - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(sqrt(2) + 2)*(-a
/c)^(1/8) + (-a/c)^(1/4))/a + 1/16*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*s
qrt(sqrt(2) + 2)*(-a/c)^(1/8))*log(x^2 - x*sqrt(sqrt(2) + 2)*(-a/c)^(1/8) +
 (-a/c)^(1/4))/a + 1/16*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2)
 + 2)*(-a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1
```

```
/4))/a - 1/16*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(-a/
c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1/4))/a
```

maple [C] time = 0.01, size = 39, normalized size = 0.12

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^{8}c-a\right)^{4}e-d\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{8}c-a\right)+x\right)}{8c\operatorname{RootOf}\left(\_Z^{8}c-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(-c*x^8+a),x)
```

```
[Out] 1/8/c*sum((-_R^4*e-d)/_R^7*ln(-_R+x),_R=RootOf(_Z^8*c-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ex^{4}+d}{cx^{8}-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(-c*x^8+a),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^4 + d)/(c*x^8 - a), x)
```

mupad [B] time = 2.72, size = 2438, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a - c*x^8),x)
```

```
[Out] (atan((a^3*e^6*x + c^3*d^6*x - a*c^2*d^4*e^2*x - a^2*c*d^2*e^4*x + (2*d*e*x
*(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a
^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*((a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^
2*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*
d*e^4*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e
 + 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^
2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c
^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4 - (atan((a*c^
2*d^4*e^2*x - c^3*d^6*x - a^3*e^6*x + a^2*c*d^2*e^4*x + (2*d*e*x*(a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2)))/(a^3*c^2))/(a*c^3*d^5*(-(a^2*e^4*(a^7*c
^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 6
*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a^2*e^4*(a^7*
c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 +
6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^2*(-(a^2*
e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3
*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4*(-
(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^
5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*(-(a^2*e^4*
(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c^3*d*e
^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4))/4 - atan((a^3*e^6*x*1
i + c^3*d^6*x*1i - a*c^2*d^4*e^2*x*1i - a^2*c*d^2*e^4*x*1i + (d*e*x*(a^2*e^
4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d
*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*((a^2*e^4*(
```

```
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*((a^2*e^4*(
a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^
3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3*e^2*((a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*
c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^3*c*d*e^4
*((a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*
a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))*((a^2*e^4
*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) + 4*a^4*c^4*d^3*e + 4*a^5*c^3*d*
e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i + atan((a*c^2
*d^4*e^2*x*1i - c^3*d^6*x*1i - a^3*e^6*x*1i + a^2*c*d^2*e^4*x*1i + (d*e*x*(
a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5
*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))*2i)/(a^3*c^2))/(a*c^3*d^5*(-(a^
2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) + a^5*c^3*e*(-(a
^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4*a^5*
c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(5/4) + 2*a^2*c^2*d^3
*e^2*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e
 - 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4) - 3*a^
3*c*d*e^4*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*
d^3*e - 4*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(a^7*c^5))^(1/4)))
*(-(a^2*e^4*(a^7*c^5)^(1/2) + c^2*d^4*(a^7*c^5)^(1/2) - 4*a^4*c^4*d^3*e - 4
*a^5*c^3*d*e^3 + 6*a*c*d^2*e^2*(a^7*c^5)^(1/2))/(4096*a^7*c^5))^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(-c*x**8+a),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8} \, dx$

Optimal. Leaf size=791

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{2\ldots\right.}{8\sqrt{\ldots}}$$

Rubi [A] time = 0.86, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*S
qrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqr
t[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] - 2*Sq
rt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt
[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b
 + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sq
rt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*
Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b
 + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sq
rt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt
[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt
[d]*Sqrt[e] - Sqrt[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*
Sqrt[e] - Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[
-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b +
 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]*x + Sq
rt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}\,x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}\,x}{\sqrt{e}}}\,\cdots}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b}}\cdots}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}\,x}{\sqrt{e}}+x^2}\cdots}{8\sqrt{d}\sqrt{e}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\,x+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\,x\cdots\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{-b+2de}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{-b+2de}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\cdots}}{\sqrt{2\sqrt{d}\sqrt{e}\cdots}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\cdots}}$$

Mathematica [C] time = 0.04, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2+\#1^4b+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b
*#1^3 + 2*e^2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^4}{d^2 + bx^4 + e^2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8), x]
```

fricas [B] time = 1.86, size = 3059, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/
(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d
^3*e + b^2*d^2)))*arctan(-1/4*(2*sqrt(1/2)*((8*d^5*e^3 + 12*b*d^4*e^2 + 6*b
^2*d^3*e + b^3*d^2)*x*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d
^5*e + b^3*d^4)) - (4*d^2*e^2 + 4*b*d*e + b^2)*x)*sqrt(-((4*d^4*e^2 + 4*b*d
^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
 b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)) + (4*d^2*e^2 + 4*b*d*e +
 b^2 - (8*d^5*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)
/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*sqrt(-((4*d^4*e^2 + 4
*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5
*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))*sqrt((2*e^2*x^2 + sq
rt(1/2)*(2*b*d*e + b^2 - (8*d^5*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)
*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*sqr
t(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d
^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))/e
^2))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e -
 b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4
*b*d^3*e + b^2*d^2)))/e) + sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^
2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)
) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*arctan(-1/4*(2*sqrt(1/2)*((8*d^5
*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*x*sqrt(-(2*d*e - b)/(8*d^7*e^3
 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + (4*d^2*e^2 + 4*b*d*e + b^2)*x)*
sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8
*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3
*e + b^2*d^2)))*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(
8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^
3*e + b^2*d^2)) - (4*d^2*e^2 + 4*b*d*e + b^2 + (8*d^5*e^3 + 12*b*d^4*e^2 +
6*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*
d^5*e + b^3*d^4)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*s
qrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(
4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*
sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/
(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))*sqrt((2*e^2*x^2 + sqrt(1/2)*(2*b*d*e + b
^2 + (8*d^5*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)/(
8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*sqrt(((4*d^4*e^2 + 4*b*
d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e
+ b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))/e^2))/e) + 1/4*sqrt(sq
rt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e
^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b
^2*d^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2
*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqr
t(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^
3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^
```

```
2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqr
t(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*
d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 + 4*b*d^
3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt
(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d
^4*e^2 + 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*
b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*
e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e
 + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^
6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*
d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e
+ b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*
sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b
*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))
*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)
/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sq
rt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d
^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.05, size = 53, normalized size = 0.07

$$\frac{\left(\mathrm{RootOf}\left(e^2\_Z^8+b\_Z^4+d^2\right)^4 e+d\right)\ln\left(-\mathrm{RootOf}\left(e^2\_Z^8+b\_Z^4+d^2\right)+x\right)}{8\,\mathrm{RootOf}\left(e^2\_Z^8+b\_Z^4+d^2\right)^7 e^2+4\,\mathrm{RootOf}\left(e^2\_Z^8+b\_Z^4+d^2\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*e^2+_Z^4*b+
d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4+d}{e^2x^8+bx^4+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

mupad [B] time = 3.83, size = 10409, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(b*x^4 + d^2 + e^2*x^8),x)
```

```
[Out] 2*atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e
^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e
```

```
)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)
))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b
^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 -
 65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2
*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d
^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 19660
8*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)
^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3
*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 + 256*b*d^6*e
^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d
*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*
d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 - 4*b^4*
d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((b - 2*d*e)*(b +
2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b
^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 327
68*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 +
20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d
^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*
b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^
13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*
e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2
)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4
*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2
*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*
e^2)))^(1/4))/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^
2*d^4*e^12) + (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*
b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d
^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 -
 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6
*e^12 - 65536*b^2*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) +
4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^
5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 409
6*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11
+ 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b +
 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*
b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 + 256*
b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(
b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 +
 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^1
3 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 + ((b - 2*
d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*
e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d
^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (
-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^
9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*
b^2*d^8*e^13)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 +
64*b^3*d^4*e^11)*1i)*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e
^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*1i))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*
b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*
e^3 + 24*b^2*d^4*e^2)))^(1/4) - atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 +
```

```
24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^
14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d
^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((b - 2*d*e)*
(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
+ 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 -
262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^1
0 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*(-(b^3
 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d
^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 25
6*d^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 +
((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2
+ 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x
*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(
b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^
4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(
(x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9
 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2
*d^7*e^13) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b
^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^
4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4
096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*
e^12 - 196608*b^2*d^8*e^13))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4
*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5
*e^3 + 24*b^2*d^4*e^2)))^(3/4) + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3
*e^10 + 64*b^3*d^4*e^11))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*
d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^
3 + 24*b^2*d^4*e^2)))^(1/4)*1i)/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^
3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2)
+ 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*
d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*b*d^8*e^14 +
1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^
11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + (-(b^3 + ((b - 2*d*e)*(b +
2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b
^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 26214
4*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 4
9152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*(-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7
*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 + ((b -
 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) - (x*(32*b*d
^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 + ((
b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(6553
6*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240
*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^1
3) - (-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*
d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 1
96608*b^2*d^8*e^13))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e
^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(3/4) + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 +
64*b^3*d^4*e^11))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(1/4)))*(-(b^3 + ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*
e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
```

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24*b^2*d^4*e^2)))^(1/4)*2i + atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*
b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2
) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^
(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^
14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d
^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 -
32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10
 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 -
((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2
+ 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d
^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 - ((b
 - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 1
6*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x*(3
2*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3
 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d
^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-
(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b
^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*
(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9
 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b
^2*d^8*e^13) + x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 20
48*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^
12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*
d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^
3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^
10 - 64*b^3*d^4*e^11))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2
*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 +
 24*b^2*d^4*e^2)))^(1/4)*1i)/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d
^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4
*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5
*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2)
 + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b
*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 +
4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^
11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 - 32768
*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20
480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b -
 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^
14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*(-(b^3 - ((b - 2*
d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) - (x*(32*b*d^5*
e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (-(b^3 - ((b -
 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - (
(b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 +
 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*
d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152
*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e
^13) + x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d
^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 655
36*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
+ 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 + 256*b*d^6*e^13 + 16*b^4*d^3*e^10 - 64*
b^3*d^4*e^11))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4
*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*
d^4*e^2)))^(1/4))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2
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 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*
b^2*d^4*e^2)))^(1/4)*2i - 2*atan(((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b
^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2)
 + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b
*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^1
4 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^
6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i + x*(65536*d^9*e^15
- 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^
10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3
- ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^
2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i +
256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*(-(b
^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4
*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) +
(x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) - (
-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^
4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196
608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*
e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^
3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2
) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16
*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(
1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e +
32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4))/((x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10
 + 24*b^3*d^3*e^11 - 48*b^2*d^4*e^12) + (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5
)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e
 + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*
e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d
^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*
d^9*e^14 + 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152
*b^4*d^6*e^11 + 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i + x*(65536*d^
9*e^15 - 32768*b*d^8*e^14 + 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5
*d^4*e^10 + 20480*b^4*d^5*e^11 + 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*
(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*
(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4
)*1i + 256*d^7*e^14 - 256*b*d^6*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1
i)*(-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4)*1i - (x*(32*b*d^5*e^13 - 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 - 48*b^2*d^4
*e^12) - (-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d
*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^
2)))^(1/4)*(((-(b^3 - ((b - 2*d*e)*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b
^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^
4*e^2)))^(1/4)*(262144*d^10*e^15 - 262144*b*d^9*e^14 + 4096*b^7*d^3*e^8 - 4
096*b^6*d^4*e^9 - 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 + 196608*b^3*d^7*
e^12 - 196608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 - 32768*b*d^8*e^14 + 102
4*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 - 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11
+ 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*(-(b^3 - ((b - 2*d*e)*(b + 2*d*
e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^3*d
^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 - 256*b*d^6
*e^13 - 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*(-(b^3 - ((b - 2*d*e)*(b + 2
*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 + 8*b^
```

```
3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i))*(-(b^3 - ((b - 2*d*e)
*(b + 2*d*e)^5)^(1/2) + 4*b*d^2*e^2 + 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
 + 8*b^3*d^3*e + 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
```

sympy [A] time = 8.50, size = 136, normalized size = 0.17

$$\operatorname{RootSum}\left(t^8\left(65536b^4d^2+524288b^3d^3e+1572864b^2d^4e^2+2097152bd^5e^3+1048576d^6e^4\right)+t^4\left(256b^3+1024b^2de+1024bd^2e^2\right)+e^2,\left(t\mapsto t\log\left(x+\frac{1024t^5b^2d^2+4096t^5bd^3e+4096t^5d^4e^2+4tb+4tde}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8+b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 + 524288*b**3*d**3*e + 1572864*b**2*d**4*e**
2 + 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(256*b**3 + 1024*b**2*
d*e + 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d**2
 + 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 + 4*_t*b + 4*_t*d*e)/e)))
```

3.6 $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

Optimal. Leaf size=791

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de}\right.}{8\sqrt{d}}$$

Rubi [A] time = 0.81, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sq
rt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[
2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]] - 2*Sqrt[e
]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*S
qrt[e] - Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e -
f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sq
rt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] +
 Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]]
/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) - Log[Sqrt[d] - Sqrt
[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[2*d*e - f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] -
Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e - f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + Log[Sq
rt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + Sqrt[e]*x^2]/(8*Sqrt[
d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}\,x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+\ldots}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2d\ldots}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}} + \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}\,x}{\sqrt{e}}+x^2\ldots}}{8\sqrt{d}\sqrt{e}}$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\,x+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}\ldots\right.}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-\ldots}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de-f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\ldots}}{\sqrt{2\sqrt{d}\ldots}}\right.}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}\ldots}}$$

Mathematica [C] time = 0.04, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2+\#1^4f+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2+\#1^3f}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(f
*#1^3 + 2*e^2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^4}{d^2 + fx^4 + e^2x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8), x]
```

fricas [B] time = 1.80, size = 3059, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/
(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3
*e*f + d^2*f^2)))*arctan(-1/4*(2*sqrt(1/2)*((8*d^5*e^3 + 12*d^4*e^2*f + 6*d
^3*e*f^2 + d^2*f^3)*x*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e
*f^2 + d^4*f^3)) - (4*d^2*e^2 + 4*d*e*f + f^2)*x)*sqrt(-((4*d^4*e^2 + 4*d^3
*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 +
 d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)) + (4*d^2*e^2 + 4*d*e*f +
 f^2 - (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*sqrt(-(2*d*e - f)
/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*sqrt(-((4*d^4*e^2 + 4
*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f
^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2))*sqrt((2*e^2*x^2 + sq
rt(1/2)*(2*d*e*f + f^2 - (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)
*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*sqr
t(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6
*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))/e
^2))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e -
 f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4
*d^3*e*f + d^2*f^2)))/e) + sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^
2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)
) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*arctan(-1/4*(2*sqrt(1/2)*((8*d^5
*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*x*sqrt(-(2*d*e - f)/(8*d^7*e^3
 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + (4*d^2*e^2 + 4*d*e*f + f^2)*x)*
sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8
*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e
*f + d^2*f^2)))*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(
8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*
e*f + d^2*f^2)) - (4*d^2*e^2 + 4*d*e*f + f^2 + (8*d^5*e^3 + 12*d^4*e^2*f +
6*d^3*e*f^2 + d^2*f^3)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*
e*f^2 + d^4*f^3)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*s
qrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(
4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*
sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/
(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2))*sqrt((2*e^2*x^2 + sqrt(1/2)*(2*d*e*f + f
^2 + (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*sqrt(-(2*d*e - f)/(
8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*sqrt(((4*d^4*e^2 + 4*d^
3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2
+ d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))/e^2))/e) + 1/4*sqrt(sq
rt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e
^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d
^2*f^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2
*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqr
t(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^
3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^
```

```
2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqr
t(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*
d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 + 4*d^3*
e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 +
d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt
(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d
^4*e^2 + 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*
d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^
2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e
 + (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*
e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^
3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2
+ d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*
sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d
^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))
*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)
/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sq
rt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6
*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2))))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.05, size = 53, normalized size = 0.07

$$\frac{\left(\mathrm{RootOf}\left(e^2\_Z^8+f\_Z^4+d^2\right)^4 e+d\right)\ln\left(-\mathrm{RootOf}\left(e^2\_Z^8+f\_Z^4+d^2\right)+x\right)}{8\,\mathrm{RootOf}\left(e^2\_Z^8+f\_Z^4+d^2\right)^7 e^2+4\,\mathrm{RootOf}\left(e^2\_Z^8+f\_Z^4+d^2\right)^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*f)*ln(-_R+x),_R=RootOf(_Z^8*e^2+_Z^4*f+
d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4+d}{e^2x^8+fx^4+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

mupad [B] time = 4.03, size = 10411, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(f*x^4 + d^2 + e^2*x^8),x)
```

```
[Out] 2*atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f
^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^
```

```
2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048
*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3
 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2
*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f
 + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3
*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196
608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*
e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e
*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 + 256*d^6*e
^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^
10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d
*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*
e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + ((-(f^3 + ((f - 2*d*e)*(f
 + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8
*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 3
2768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5
 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144
*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 4915
2*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13
*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*
f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f
^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e
^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*
e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e
*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4))/(((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f +
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7
 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e
^12*f^3 - 65536*d^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2)
+ 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4
096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^
4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f
 + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 +
8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 + 25
6*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4
*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(
f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 +
 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - ((-(f^3 + ((f -
2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^
9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4
*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) +
 (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*
f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 19660
8*d^8*e^13*f^2)*1i)*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*
f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24
*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4
+ 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3
 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2
*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 2
4*d^4*e^2*f^2)))^(1/4)*1i))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*
d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^
3*f + 24*d^4*e^2*f^2)))^(1/4) - atan((((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)
```

```
^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
+ 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^
14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5
*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 + ((f - 2*d*e
)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15
- 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*
f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*(-(f
^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*
d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) -
256*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*
d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i + ((
-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(
16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)
*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f
^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d
^7*e^13*f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 -
 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^
12*f^3 - 196608*d^8*e^13*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) +
 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(3/4) + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e
^10*f^4 + 64*d^4*e^11*f^3) - x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^1
1*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^
2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*
f + 24*d^4*e^2*f^2)))^(1/4)*1i)/(((-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2
) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*
d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 - 32768*d^8*e^14*f
+ 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11
*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + (-(f^3 + ((f - 2*d*e)*(f
+ 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8
*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262
144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 +
 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*(-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d
^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*d^5*e
^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 + ((f -
 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) - ((-(f^3 +
((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65
536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 102
40*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*
f^2) - (-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^
4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 -
 196608*d^8*e^13*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e
^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(3/4) + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4
+ 64*d^4*e^11*f^3) - x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 -
48*d^4*e^12*f^2))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f
+ 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d
^4*e^2*f^2)))^(1/4)))*(-(f^3 + ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^
2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f +
```

24*d^4*e^2*f^2)))^(1/4)*2i - atan((((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1
/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5
)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^
14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6
*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2) + x*(65536*d^9*e^15
- 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f
^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3
- ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6
*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256
*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*d^5
*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f
 - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - ((-(f
^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*
d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((
(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*
(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4
)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f
^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608
*d^8*e^13*f^2) - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 -
2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12
*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*
d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^
3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10
*f^4 - 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f
^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e
^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(1/4)*1i)/(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) +
 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/
2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32
*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f
+ 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11
*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2) + x*(65536*d^9*e^15 - 327
68*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 +
20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f
 - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*
e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) - x*(32*d^5*e^13
*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*
d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2
*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + ((-(f^3 - ((f
 - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 -
 ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*
e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(26214
4*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 491
52*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^1
3*f^2) - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3
*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 6
5536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*
f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24
*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 + 256*d^6*e^13*f + 16*d^3*e^10*f^4 - 6
4*d^4*e^11*f^3) + x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*
d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f

+ 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*
d^4*e^2*f^2)))^(1/4)*2i - 2*atan((((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/
2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32
*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)
^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3
+ 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^1
4*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*
e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i + x*(65536*d^9*e^1
5 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10
*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^
3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d
^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i
+ 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i + x
*(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f
^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*
d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) -
((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*
e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 1
96608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^
8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*
d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1
/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f -
16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f - 4*d^2*e^10*f^4 +
24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(
1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 +
32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4))/(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)
^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f
^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*
d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3
*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*
d^9*e^14*f + 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 491
52*d^6*e^11*f^4 + 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i + x*(65536*
d^9*e^15 - 32768*d^8*e^14*f + 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d
^4*e^10*f^5 + 20480*d^5*e^11*f^4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2)
)*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(51
2*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3
/4)*1i + 256*d^7*e^14 - 256*d^6*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)
1i + x(32*d^5*e^13*f - 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2
))*(-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
1/4)*1i + ((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e
f^2)/(512(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4)*(((-(f^3 - ((f - 2*d*e)*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)*(262144*d^10*e^15 - 262144*d^9*e^14*f + 4096*d^3*e^8*f^7 -
4096*d^4*e^9*f^6 - 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 + 196608*d^7*e^
12*f^3 - 196608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 - 32768*d^8*e^14*f + 1
024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 - 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^
4 + 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2*
d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^3
*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 - 256*d^6
*e^13*f - 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f - 4*d^2*
e^10*f^4 + 24*d^3*e^11*f^3 - 48*d^4*e^12*f^2))*(-(f^3 - ((f - 2*d*e)*(f + 2
*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 + 8*d^

```
3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i))*(-(f^3 - ((f - 2*d*e)
*(f + 2*d*e)^5)^(1/2) + 4*d^2*e^2*f + 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
 + 8*d^3*e*f^3 + 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)
```

sympy [**A**] time = 7.14, size = 136, normalized size = 0.17

$$\operatorname{RootSum}\left(t^8\left(1048576d^6e^4+2097152d^5e^3f+1572864d^4e^2f^2+524288d^3ef^3+65536d^2f^4\right)+t^4\left(1024d^2e^2f+1024def^2+256f^3\right)+e^2,\left(t\mapsto t\log\left(x+\frac{4096t^5d^4e^2+4096t^5d^3ef+1024t^5d^2f^2+4tde+4tf}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8+f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 + 2097152*d**5*e**3*f + 1572864*d**4*e**2*
f**2 + 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(1024*d**2*e**2*f + 10
24*d*e*f**2 + 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e**2
 + 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e + 4*_t*f)/e)))
```

3.7 $\int \frac{d+ex^4}{d^2-bx^4+e^2x^8} \, dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}-\sqrt{b+}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}-}}$$

Rubi [A] time = 0.42, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1419, 1093, 207, 203}

$$-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2} \sqrt{b-2de} \sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e
]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]])) - (
Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]]
)/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]) - (Sqrt
[e]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]])/(
Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]) - (Sqrt[e]
*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]])/(Sqr
t[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}\,x^2}{e}+x^4}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e}-\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}} - \frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e}-\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}} + \frac{\int \frac{1}{-\frac{\sqrt{b-2de}}{2e}+\frac{\sqrt{b+2de}}{2e}+x^2}\,dx}{2\sqrt{b-2de}} - \frac{\int \frac{1}{\frac{\sqrt{b-2de}}{2e}+\frac{\sqrt{b+}}{2}}}{2\sqrt{b-}}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}} - \frac{}{\sqrt{2}}$$

Mathematica [C] time = 0.04, size = 69, normalized size = 0.20

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2-\#1^4b+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2-\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 - b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-
(b*#1^3) + 2*e^2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8), x]
```

fricas [B] time = 1.71, size = 3048, normalized size = 8.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/
(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)))*arctan(-1/4*(2*sqrt(1/2)*((8*d^5*e^3 - 12*b*d^4*e^2 + 6*b
^2*d^3*e - b^3*d^2)*x*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d
^5*e - b^3*d^4)) - (4*d^2*e^2 - 4*b*d*e + b^2)*x)*sqrt(-((4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
 b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)) + (4*d^2*e^2 - 4*b*d*e +
 b^2 - (8*d^5*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)
/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sqrt(-((4*d^4*e^2 - 4
*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5
*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))*sqrt((2*e^2*x^2 - sq
rt(1/2)*(2*b*d*e - b^2 + (8*d^5*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)
*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sqr
t(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d
^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))/e
^2))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e +
```

```
 b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4
*b*d^3*e + b^2*d^2)))/e) + sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^
2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)
) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*arctan(-1/4*(2*sqrt(1/2)*((8*d^5
*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*x*sqrt(-(2*d*e + b)/(8*d^7*e^3
 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + (4*d^2*e^2 - 4*b*d*e + b^2)*x)*
sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8
*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3
*e + b^2*d^2)))*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(
8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^
3*e + b^2*d^2)) - (4*d^2*e^2 - 4*b*d*e + b^2 + (8*d^5*e^3 - 12*b*d^4*e^2 +
6*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*
d^5*e - b^3*d^4)))*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*s
qrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(
4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*
sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/
(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))*sqrt((2*e^2*x^2 - sqrt(1/2)*(2*b*d*e - b
^2 - (8*d^5*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)/(
8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sqrt(((4*d^4*e^2 - 4*b*
d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e
- b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))/e^2))/e) + 1/4*sqrt(sq
rt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^
3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^
2*d^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt
(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3
- 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*
d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-
(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4
*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 - 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3
*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2
*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d
^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
 b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e -
(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e
^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^
3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e -
b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sq
rt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*
d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*
log(e*x - 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/
(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqr
t(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d
^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.03, size = 55, normalized size = 0.16

$$\frac{\left(\mathrm{RootOf}\left(e^{2}\_Z^{8}-b\_Z^{4}+d^{2}\right)^{4}e+d\right)\ln\left(-\mathrm{RootOf}\left(e^{2}\_Z^{8}-b\_Z^{4}+d^{2}\right)+x\right)}{8\,\mathrm{RootOf}\left(e^{2}\_Z^{8}-b\_Z^{4}+d^{2}\right)^{7}e^{2}-4\,\mathrm{RootOf}\left(e^{2}\_Z^{8}-b\_Z^{4}+d^{2}\right)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*e^2-_Z^4*b+
d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4+d}{e^2x^8-bx^4+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

mupad [B] time = 4.03, size = 10337, normalized size = 29.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x)
```

```
[Out] 2*atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e
^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)
/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2))
)^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^
6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 -
65536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e
^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3
*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*
b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))
^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
- 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 256*b*d^6*e^13
 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e
))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*
e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 + 4*b^4*d^2*
e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d
*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^
3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*
d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480
*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*
d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^
15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^
5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i
)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512
*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/
4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*
1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
1/4))/((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^
12) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
```

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(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6
*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 6
5536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^
2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24
*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*
e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b
^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^
(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e -
 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i - 256*d^7*e^14 - 256*b*d^6*e^13
+ 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e)
)^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
 - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^13 + 4*b^4*d^
2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 + ((b - 2*d*e)^5*(b + 2
*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*
d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*
b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 204
80*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b -
2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*
e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*
d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*
1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(5
12*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(
3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11
)*1i)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(1/4)*1i))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2
*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*
e^2)))^(1/4) - atan(((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 +
 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2
 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*
b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*
e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^
3*d^6*e^12 - 65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2
) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 -
 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e
^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*((b^3 + ((b - 2*d*e)^5*(b
 + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*
b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d
^6*e^13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 + ((b - 2*d*e)^5*(b + 2
*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*
d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x*(32*b*d^5*e^13 + 4*b
^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(
b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8
*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 3
2768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10
+ 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) - ((b^3 + (
(b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 +
 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*
d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152
*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e
^13))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/
(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))
^(3/4) + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)
)*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512
*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/
4)*1i)/((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e
```

^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)
/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2))
)^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^
6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 -
65536*b^2*d^7*e^13) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e
^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3
*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*
b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1
/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 3
2*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^13 + 16*
b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2)
+ 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*
d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4) - (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24
*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 + ((b - 2*d*e)^5*(b + 2*d*e))^(1/2
) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*
b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*b*d^8*e^14
- 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*
e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13) - ((b^3 + ((b - 2*d*e)^5*(b
 + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*
b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 2621
44*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 +
49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13))*((b^3 + ((
b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) + 256*d^7
*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11))*((b^3 + ((b -
2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)))*((b^3 + ((b
 - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 1
6*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*2i - atan(
((x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) +
((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(
b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(51
2*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1
/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4
*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 1966
08*b^2*d^8*e^13) + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8
- 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^
6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4
*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5
*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^13 + 16*b^4*d^3
*e^10 + 64*b^3*d^4*e^11))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d
^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3
 + 24*b^2*d^4*e^2)))^(1/4)*1i + (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3
*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) +
4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^
5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2)
 + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b
*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 -
4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^
11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 + 32768
*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20
480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b -
2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d
^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^1
4 - 256*b*d^6*e^13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 - ((b - 2*d*
e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e
^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i)/((x*(32*b*d^5

```
*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) + ((b^3 - ((b -
 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((
b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 +
16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d
^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*
b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^
13) + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^
3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 6553
6*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 -
4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2
*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^13 + 16*b^4*d^3*e^10 + 64*b^
3*d^4*e^11))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^
2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4
*e^2)))^(1/4) - (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b
^2*d^4*e^12) - ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*
b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d
^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 -
 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^
2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8
 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*
d^7*e^12 - 196608*b^2*d^8*e^13) - x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 10
24*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11
 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*
d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d
^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4) - 256*d^7*e^14 - 256*b*d^6*e^
13 + 16*b^4*d^3*e^10 + 64*b^3*d^4*e^11))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e)
)^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e
 - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)))*((b^3 - ((b - 2*d*e)^5*(b + 2*d
*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^
3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*2i - 2*atan(((x*(32*b*d^5*e^13
 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 - ((b - 2*d*
e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e
^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2
*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^
6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e
^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d
^5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1
i + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*
e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*
b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*
b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d
^4*e^2)))^(3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b
^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 -
4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2
*d^4*e^2)))^(1/4) + (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 +
48*b^2*d^4*e^12) + ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2
- 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b
^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e
^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 2
4*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d^3
*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 196608*
b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 + 32768*b*d^8*e^
14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d
^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*
(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 -
8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 + 25
6*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e)^
5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
```

```
- 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4))/((x*(32*b*d^5*e^13
+ 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11 + 48*b^2*d^4*e^12) - ((b^3 - ((b - 2*d*e
)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^
4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*
d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6
*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^
15 + 262144*b*d^9*e^14 - 4096*b^7*d^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^
5*e^10 + 49152*b^4*d^6*e^11 - 196608*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i
 + x*(65536*d^9*e^15 + 32768*b*d^8*e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e
^9 + 10240*b^5*d^4*e^10 + 20480*b^4*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b
^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b
^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^
4*e^2)))^(3/4)*1i + 256*d^7*e^14 + 256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^
3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4
*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*
d^4*e^2)))^(1/4)*1i - (x*(32*b*d^5*e^13 + 4*b^4*d^2*e^10 + 24*b^3*d^3*e^11
+ 48*b^2*d^4*e^12) + ((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^
2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24
*b^2*d^4*e^2)))^(1/4)*((((b^3 - ((b - 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2
*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 +
 24*b^2*d^4*e^2)))^(1/4)*(262144*d^10*e^15 + 262144*b*d^9*e^14 - 4096*b^7*d
^3*e^8 - 4096*b^6*d^4*e^9 + 49152*b^5*d^5*e^10 + 49152*b^4*d^6*e^11 - 19660
8*b^3*d^7*e^12 - 196608*b^2*d^8*e^13)*1i - x*(65536*d^9*e^15 + 32768*b*d^8*
e^14 - 1024*b^7*d^2*e^8 - 2048*b^6*d^3*e^9 + 10240*b^5*d^4*e^10 + 20480*b^4
*d^5*e^11 - 32768*b^3*d^6*e^12 - 65536*b^2*d^7*e^13))*((b^3 - ((b - 2*d*e)^
5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^4
- 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(3/4)*1i + 256*d^7*e^14 +
256*b*d^6*e^13 - 16*b^4*d^3*e^10 - 64*b^3*d^4*e^11)*1i)*((b^3 - ((b - 2*d*e
)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*d^6*e^
4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)*1i))*((b^3 - ((b -
 2*d*e)^5*(b + 2*d*e))^(1/2) + 4*b*d^2*e^2 - 4*b^2*d*e)/(512*(b^4*d^2 + 16*
d^6*e^4 - 8*b^3*d^3*e - 32*b*d^5*e^3 + 24*b^2*d^4*e^2)))^(1/4)
```

sympy [A] time = 8.25, size = 136, normalized size = 0.39

$$\operatorname{RootSum}\left(t^{8}\left(65536b^{4}d^{2}-524288b^{3}d^{3}e+1572864b^{2}d^{4}e^{2}-2097152bd^{5}e^{3}+1048576d^{6}e^{4}\right)+t^{4}\left(-256b^{3}+1024b^{2}de-1024bd^{2}e^{2}\right)+e^{2},\left(t\mapsto t\log\left(x+\frac{1024t^{5}b^{2}d^{2}-4096t^{5}bd^{3}e+4096t^{5}d^{4}e^{2}-4tb+4tde}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8-b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 - 524288*b**3*d**3*e + 1572864*b**2*d**4*e**
2 - 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(-256*b**3 + 1024*b**2
*d*e - 1024*b*d**2*e**2) + e**2, Lambda(_t, _t*log(x + (1024*_t**5*b**2*d**
2 - 4096*_t**5*b*d**3*e + 4096*_t**5*d**4*e**2 - 4*_t*b + 4*_t*d*e)/e)))
```

3.8 $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

Optimal. Leaf size=751

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{\log\left(-x\sqrt{\right.}}{8\sqrt{}}$$

Rubi [A] time = 0.92, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sq
rt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[
2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]] - 2*Sqrt[e
]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*S
qrt[e] - Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e +
f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sq
rt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] +
 Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]]
/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) - Log[Sqrt[d] - Sqrt
[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sq
rt[d]*Sqrt[e] - Sqrt[2*d*e + f]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] -
Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e + f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + Log[Sq
rt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + Sqrt[e]*x^2]/(8*Sqrt[
d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}\,x^2}{e}+x^4}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}\,x^2}{e}+x^4}\,dx}{2e}$$

$$= \frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}{\sqrt{e}}+x}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,dx}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\int \frac{\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}{\sqrt{e}}-x}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\cdots}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+\cdots}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}+\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,dx}{8\sqrt{d}\sqrt{e}}+\frac{\int \frac{1}{\frac{\sqrt{d}}{\sqrt{e}}-\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}\,x}{\sqrt{e}}+x^2}\,d\cdots}{8\sqrt{d}\sqrt{e}}+\cdots$$

$$= -\frac{\log\left(\sqrt{d}-\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x+\sqrt{e}\,x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\log\left(\sqrt{d}+\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}\,x+\cdots\right.}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}+\sqrt{2de+f}}-2\sqrt{e}\,x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2d\cdots}}}{\sqrt{2\sqrt{d}\sqrt{e}+\cdots}}\right.}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}+\cdots}}\cdots$$

Mathematica [C] time = 0.04, size = 69, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2-\#1^4f+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2-\#1^3f}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 - f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(-
(f*#1^3) + 2*e^2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^4}{d^2 - fx^4 + e^2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8), x]
```

fricas [B] time = 1.62, size = 3051, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(
8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*
e*f + d^2*f^2)))*arctan(1/4*(2*sqrt(1/2)*((8*d^5*e^3 - 12*d^4*e^2*f + 6*d^3
*e*f^2 - d^2*f^3)*x*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f
^2 - d^4*f^3)) + (4*d^2*e^2 - 4*d*e*f + f^2)*x)*sqrt(((4*d^4*e^2 - 4*d^3*e*
f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^
4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)) - (4*d^2*e^2 - 4*d*e*f + f^
2 + (8*d^5*e^3 - 12*d^4*e^2*f + 6*d^3*e*f^2 - d^2*f^3)*sqrt(-(2*d*e + f)/(8
*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)))*sqrt(((4*d^4*e^2 - 4*d^3
*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 -
 d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2))*sqrt((2*e^2*x^2 - sqrt(1
/2)*(2*d*e*f - f^2 - (8*d^5*e^3 - 12*d^4*e^2*f + 6*d^3*e*f^2 - d^2*f^3)*sqr
t(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)))*sqrt(((
4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*
f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))/e^2))*
sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8
*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e
*f + d^2*f^2)))/e) + sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2
)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f
)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*arctan(1/4*(2*sqrt(1/2)*((8*d^5*e^3 -
 12*d^4*e^2*f + 6*d^3*e*f^2 - d^2*f^3)*x*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*
d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - (4*d^2*e^2 - 4*d*e*f + f^2)*x)*sqrt(s
qrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*
e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f +
d^2*f^2)))*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^
7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f
+ d^2*f^2)) + (4*d^2*e^2 - 4*d*e*f + f^2 - (8*d^5*e^3 - 12*d^4*e^2*f + 6*d^
3*e*f^2 - d^2*f^3)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^
2 - d^4*f^3)))*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt
(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d
^4*e^2 - 4*d^3*e*f + d^2*f^2)))*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sq
rt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4
*d^4*e^2 - 4*d^3*e*f + d^2*f^2))*sqrt((2*e^2*x^2 - sqrt(1/2)*(2*d*e*f - f^2
 + (8*d^5*e^3 - 12*d^4*e^2*f + 6*d^3*e*f^2 - d^2*f^3)*sqrt(-(2*d*e + f)/(8*
d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)))*sqrt(-((4*d^4*e^2 - 4*d^3
*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 -
 d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))/e^2))/e) + 1/4*sqrt(sqr
t(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3
 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2
*f^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d
*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(
1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 -
 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f
```

```
^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(
2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*
e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 - 4*d^3*e*f
+ d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*
f^3)) - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^
2 - 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*
e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 -
d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e - (
4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*
f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e
*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d
^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqr
t(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6
*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*l
og(e*x - 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(
8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt
(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*
e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2))))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.03, size = 55, normalized size = 0.07

$$\frac{\left(\mathrm{RootOf}\left(e^2\_Z^8-f\_Z^4+d^2\right)^4 e+d\right)\ln\left(-\mathrm{RootOf}\left(e^2\_Z^8-f\_Z^4+d^2\right)+x\right)}{8\,\mathrm{RootOf}\left(e^2\_Z^8-f\_Z^4+d^2\right)^7 e^2-4\,\mathrm{RootOf}\left(e^2\_Z^8-f\_Z^4+d^2\right)^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*f)*ln(-_R+x),_R=RootOf(_Z^8*e^2-_Z^4*f+
d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4+d}{e^2x^8-fx^4+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

mupad [B] time = 4.20, size = 10343, normalized size = 13.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x)
```

```
[Out] 2*atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^
2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2
```

```
)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*
d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3
- 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e
))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^
3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 256*d^6*e^13
*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*
f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e
))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^
3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) + (((f^3 + ((f - 2*d*e)^5*(f + 2
*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*
e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*
d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 204
80*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*
e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*
e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*
1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3
)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^
2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(5
12*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(
1/4))/((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d
^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 -
 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^
2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^
8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608
*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i - 256*d^7*e^14 - 256*d^6*e^13*
f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f
^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i - (((f^3 + ((f - 2*d*e)^5*(f +
 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^
3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 3276
8*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 2
0480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f
- 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
 d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^1
0*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^
5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2
)*1i)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f
^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*
f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/
(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))
^(1/4)*1i))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e
*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*
f^2)))^(1/4) - atan(((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2
```

```
*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 2
4*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^
8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*
d^6*e^12*f^3 - 65536*d^7*e^13*f^2) + ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1
/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f
 - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^1
1*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*((f^3 + ((f - 2*d*e)^5*
(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d
^6*e^13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e
^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2
*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*
e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i + (((f^3 + ((f - 2*d*e)^5
*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 +
 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^
5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) - ((f^3 +
 ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*
e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(26214
4*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 491
52*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^1
3*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2
)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)
))^(3/4) + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^
3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2)
)*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*1i)/((((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^
2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2
)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*
d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3
- 65536*d^7*e^13*f^2) + ((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^
(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 -
 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^13*f + 1
6*d^3*e^10*f^4 + 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*
d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1/2)
+ 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4) - (((f^3 + ((f - 2*d*e)^5*(f + 2*d*e))^(1
/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 3
2*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((x*(65536*d^9*e^15 + 32768*d^8*e^14*
f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^
11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2) - ((f^3 + ((f - 2*d*e)^5*
(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 -
8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 26
2144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5
+ 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2))*((f^3 +
((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) + 256*d
^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3) + x*(32*d^5*e
^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 + ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)))*((f^3 + ((f
 - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*2i - atan(
```

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((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512
*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/
4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(
512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^
(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e
^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 19
6608*d^8*e^13*f^2) + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^
7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*
e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) +
 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5
*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^13*f + 16*d^3*e
^10*f^4 + 64*d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^1
1*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2
*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f
 + 24*d^4*e^2*f^2)))^(1/4)*1i - (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2)
+ 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^
5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/
2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32
*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f
- 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11
*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2) - x*(65536*d^9*e^15 + 327
68*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 +
20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f
- 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
 d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e
^14 - 256*d^6*e^13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3) - x*(32*d^5*e^13*
f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*
e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f
^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i)/((((f^3 - ((f
 - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4
+ d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 -
((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e
^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144
*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 4915
2*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13
*f^2) + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*
e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65
536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f
- 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d
^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^13*f + 16*d^3*e^10*f^4 + 64*
d^4*e^11*f^3) + x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^
4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*
e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2
*f^2)))^(1/4) + (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f -
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f
 - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*
d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f
^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^
7*e^12*f^3 - 196608*d^8*e^13*f^2) - x*(65536*d^9*e^15 + 32768*d^8*e^14*f -
1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f
^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f +
2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3
*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4) - 256*d^7*e^14 - 256*d^6*e^
13*f + 16*d^3*e^10*f^4 + 64*d^4*e^11*f^3) - x*(32*d^5*e^13*f + 4*d^2*e^10*f
^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e)
)^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3
 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)))*((f^3 - ((f - 2*d*e)^5*(f + 2*d
*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*
```

f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*2i - 2*atan(((((f^3 - ((f - 2*
d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2
*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10
*e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5
*e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)
1i + x(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^
9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 6553
6*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f -
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64
*d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 4
8*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f -
4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4
*e^2*f^2)))^(1/4) - (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*
f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24
*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e
^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3*e
^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 19660
8*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 + 32768*d^8*e^
14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5
*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^
5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
- 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 +
256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f +
4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^
5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4
- 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4))/((((f^3 - ((f - 2*d
e)^5(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*
f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f -
2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d
^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*
e^15 + 262144*d^9*e^14*f - 4096*d^3*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*
e^10*f^5 + 49152*d^6*e^11*f^4 - 196608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*
1i + x*(65536*d^9*e^15 + 32768*d^8*e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9
*f^6 + 10240*d^4*e^10*f^5 + 20480*d^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536
*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(3/4)*1i + 256*d^7*e^14 + 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*
d^4*e^11*f^3)*1i - x*(32*d^5*e^13*f + 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48
*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4
*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*
e^2*f^2)))^(1/4)*1i + (((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^
2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f +
24*d^4*e^2*f^2)))^(1/4)*((((f^3 - ((f - 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2
*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f
+ 24*d^4*e^2*f^2)))^(1/4)*(262144*d^10*e^15 + 262144*d^9*e^14*f - 4096*d^3
*e^8*f^7 - 4096*d^4*e^9*f^6 + 49152*d^5*e^10*f^5 + 49152*d^6*e^11*f^4 - 196
608*d^7*e^12*f^3 - 196608*d^8*e^13*f^2)*1i - x*(65536*d^9*e^15 + 32768*d^8*
e^14*f - 1024*d^2*e^8*f^7 - 2048*d^3*e^9*f^6 + 10240*d^4*e^10*f^5 + 20480*d
^5*e^11*f^4 - 32768*d^6*e^12*f^3 - 65536*d^7*e^13*f^2))*((f^3 - ((f - 2*d*e
)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(3/4)*1i + 256*d^7*e^14
+ 256*d^6*e^13*f - 16*d^3*e^10*f^4 - 64*d^4*e^11*f^3)*1i + x*(32*d^5*e^13*f
+ 4*d^2*e^10*f^4 + 24*d^3*e^11*f^3 + 48*d^4*e^12*f^2))*((f^3 - ((f - 2*d*e
)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 + d^2*f^
4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)*1i))*((f^3 - ((f -

```
 2*d*e)^5*(f + 2*d*e))^(1/2) + 4*d^2*e^2*f - 4*d*e*f^2)/(512*(16*d^6*e^4 +
d^2*f^4 - 8*d^3*e*f^3 - 32*d^5*e^3*f + 24*d^4*e^2*f^2)))^(1/4)
```

sympy [A] time = 7.25, size = 136, normalized size = 0.18

$$\operatorname{RootSum}\left(t^{8}\left(1048576d^{6}e^{4}-2097152d^{5}e^{3}f+1572864d^{4}e^{2}f^{2}-524288d^{3}ef^{3}+65536d^{2}f^{4}\right)+t^{4}\left(-1024d^{2}e^{2}f+1024def^{2}-256f^{3}\right)+e^{2},\left(t\mapsto t\log\left(x+\frac{4096t^{5}d^{4}e^{2}-4096t^{5}d^{3}ef+1024t^{5}d^{2}f^{2}+4tde-4tf}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(e**2*x**8-f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 - 2097152*d**5*e**3*f + 1572864*d**4*e**2*
f**2 - 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(-1024*d**2*e**2*f + 1
024*d*e*f**2 - 256*f**3) + e**2, Lambda(_t, _t*log(x + (4096*_t**5*d**4*e**
2 - 4096*_t**5*d**3*e*f + 1024*_t**5*d**2*f**2 + 4*_t*d*e - 4*_t*f)/e)))
```

3.9 $\int \frac{1+x^4}{1+bx^4+x^8} \, dx$

Optimal. Leaf size=411

$$-\frac{\log\left(-\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(-\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}+\frac{\log\left(\sqrt{\sqrt{2-}\right.}{8\sqrt{}}$$

Rubi [A] time = 0.29, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(-\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}+\frac{\log\left(\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sq
rt[2 - b]]) - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]]/(
4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sq
rt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x
)/Sqrt[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[
2 - b]]*x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*
x + x^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x + x^2]
/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2]/(8*Sqrt
[2 + Sqrt[2 - b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
```

```
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2-b}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}-x}{1-\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}}+x}{1+\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}-x}{1-\sqrt{2+\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2-b}}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}+x}{1+\sqrt{2+\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2-b}}}$$

$$= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{2-b}}\,x+x^2}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2-b}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2-b}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2-b}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2-}}\right.}{8\sqrt{2+\sqrt{2-}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2+\sqrt{2-b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}}{\sqrt{2-\sqrt{2-}}}\right.}{4\sqrt{2-\sqrt{2-}}}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^8+\#1^4 b+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+\#1^3 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] RootSum[1 + b*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^3 + 2*
#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1+bx^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 + b*x^4 + x^8), x]
```

fricas [B] time = 1.41, size = 1443, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8))
 - b)/(b^2 + 4*b + 4)))*arctan(1/2*sqrt(1/2)*(b^2 + (b^3 + 6*b^2 + 12*b + 8
)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2
)*(b^2 + (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) +
2*b)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2
 + 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^
2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b
^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)) - 1/2*sqrt(1/2)*((b^3 + 6*b^2
 + 12*b + 8)*x*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + (b^2 + 4*b + 4)*x)*
sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8))
 - b)/(b^2 + 4*b + 4)))*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 1
2*b + 8)) - b)/(b^2 + 4*b + 4))) - sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sq
rt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*arctan(-1/2*(sq
rt(1/2)*(b^2 - (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b +
8)) + 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 + 6*b^2 + 12*b + 8)*sqr
t((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 2*b)*sqrt(-((b^2 + 4*b + 4)*sqrt((b -
 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*sqrt(-((b^2 + 4*b + 4)
*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)) + sqrt(1/2)*(
(b^3 + 6*b^2 + 12*b + 8)*x*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - (b^2 +
4*b + 4)*x)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) +
 b)/(b^2 + 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b
^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-
((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4
)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2
)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b +
8)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)
*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4)))*log(-1/2*((b
^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*sqrt(sqrt(1/2
)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 +
 4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b
^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*log(1/2*((b^2 + 4*b + 4)*sqr
t((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*
b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x) -
 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b
+ 8)) - b)/(b^2 + 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 +
6*b^2 + 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b -
2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.75Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.06, size = 42, normalized size = 0.10

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)^7+4\,\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+b*x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8+_Z^4*b+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8+bx^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

mupad [B] time = 3.68, size = 5341, normalized size = 13.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(b*x^4 + x^8 + 1),x)
```

```
[Out] - atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*
b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^
2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608
*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) +
 x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 10
24*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32
*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) +
x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*
b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b + (
(b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 +
 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b
 + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 -
 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(32768*b + 65536
*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-
(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(32*b - 48*b^2 +
24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32
*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((b - 2)*(b + 2)^5)^
(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4
*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*
b^5 + 4096*b^6 - 4096*b^7 - 262144) + x*(32768*b + 65536*b^2 - 32768*b^3 -
20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b
+ 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4
) - 256*b + 64*b^3 - 16*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-
(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
 + b^4 + 16)))^(1/4) + ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(
512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)*(b + 2)^
5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(26
2144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*
b^7 - 262144) - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5
+ 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2
+ b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 1
6*b^4 + 256) - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2
)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)))
*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*
b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) +
4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4
```

```
*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*
b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3
 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*
(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(
3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(
-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^
3 + b^4 + 16)))^(1/4) - ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/
(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)
*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^
(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^
6 - 4096*b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4
+ 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1
/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*
b^3 + 16*b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b -
 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)
))^(1/4))/(((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)*(b + 2)^5)^(
1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144
*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7
- 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 +
 2048*b^6 - 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 +
 b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4
- 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5
)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i +
 ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8
*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b
^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 19660
8*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144)*
1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6
- 1024*b^7 - 65536))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512
*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i
+ x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b + ((b - 2)*(b + 2)^5)^(1/2) +
4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i))*(-(4*b +
((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4
+ 16)))^(1/4) - atan((((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(5
12*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5
)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262
144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b
^7 - 262144) + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 +
 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 +
 b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16
*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)
^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i
 - ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 +
 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b
^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2
- 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) - x*(3
2768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^
7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) - x*(32
*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 +
 b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b - ((b -
2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16))
)^(1/4)*(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24
*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 4915
2*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144) + x*(32768*b + 65536*b^2
- 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b
- ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^
```

```
4 + 16)))^(3/4) - 256*b + 64*b^3 - 16*b^4 + 256) + x*(32*b - 48*b^2 + 24*b^
3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
24*b^2 + 8*b^3 + b^4 + 16)))^(1/4) + ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) +
4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b
 - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 1
6)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 40
96*b^6 - 4096*b^7 - 262144) - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^
4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^
(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4) - 256*
b + 64*b^3 - 16*b^4 + 256) - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - (
(b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 +
 16)))^(1/4)))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b
 + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b - ((b - 2)*(b +
2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*
(256*b + ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24
*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 4915
2*b^4 + 49152*b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b
^2 - 32768*b^3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4
*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 +
 b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b
^3 - 4*b^4))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b +
 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4) - ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) +
 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(
4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3
+ b^4 + 16)))^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152
*b^5 + 4096*b^6 - 4096*b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^
3 - 20480*b^4 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)
*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^
(3/4)*1i - 64*b^3 + 16*b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*
(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b
^3 + b^4 + 16)))^(1/4))/(((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)
/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2
)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))
^(1/4)*(262144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b
^6 - 4096*b^7 - 262144)*1i + x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4
 + 10240*b^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(
1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64
*b^3 + 16*b^4 - 256)*1i - x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b
- 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16
)))^(1/4)*1i + ((-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*
b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)*(b + 2)^
5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*(26
2144*b + 196608*b^2 - 196608*b^3 - 49152*b^4 + 49152*b^5 + 4096*b^6 - 4096*
b^7 - 262144)*1i - x*(32768*b + 65536*b^2 - 32768*b^3 - 20480*b^4 + 10240*b
^5 + 2048*b^6 - 1024*b^7 - 65536))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b
^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(3/4)*1i - 64*b^3 + 16*
b^4 - 256)*1i + x*(32*b - 48*b^2 + 24*b^3 - 4*b^4))*(-(4*b - ((b - 2)*(b +
2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2 + 8*b^3 + b^4 + 16)))^(1/4)*
1i))*(-(4*b - ((b - 2)*(b + 2)^5)^(1/2) + 4*b^2 + b^3)/(512*(32*b + 24*b^2
+ 8*b^3 + b^4 + 16)))^(1/4)
```

sympy [A] time = 3.67, size = 75, normalized size = 0.18

$$\operatorname{RootSum}\left(t^{8}\left(65536b^{4}+524288b^{3}+1572864b^{2}+2097152b+1048576\right)+t^{4}\left(256b^{3}+1024b^{2}+1024b\right)+1,\left(t\mapsto t\log\left(1024t^{5}b^{2}+4096t^{5}b+4096t^{5}+4tb+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] RootSum(_t**8*(65536*b**4 + 524288*b**3 + 1572864*b**2 + 2097152*b + 104857
6) + _t**4*(256*b**3 + 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t**
```

```
5*b**2 + 4096*_t**5*b + 4096*_t**5 + 4*_t*b + 4*_t + x)))
```

3.10 $\int \frac{1+x^4}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=451

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{4\,2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.41, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1420, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)])/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
 Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^
(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*L
og[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)
*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt
[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt
[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5
]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(
1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1420

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2
 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx = \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$=\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}$$

$$=\frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}+\frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}+\frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}}{4\sqrt{5}}$$

$$=-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\,2^{3/4}\sqrt{}}$$

$$=-\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.12

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^3 + 2*
#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.24, size = 951, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*s
qrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(2*sqrt(5) + 6)^(5
/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 2) + 1/40*sqrt(10)*(2*sqrt(5)*x - 5*x)*(2*
sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sq
rt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*
sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)
*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5)
 + 6)*(sqrt(5) - 3))*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 2)
+ 1/40*sqrt(10)*(2*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)
 + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3))
 - 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*ar
ctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)
*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(sqrt(5) +
2)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4) - 1/40*(sqrt(10)*(2*sqrt(5)*x
+ 5*x)*(-2*sqrt(5) + 6)^(5/4) + 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqr
t(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5)
+ 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(sq
rt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5) + 3)*sqr
t(-2*sqrt(5) + 6))*(sqrt(5) + 2)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4)
- 1/40*(sqrt(10)*(2*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^(5/4) - 5*(sqrt(5)*sq
rt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) - 1/80*sqrt(10
)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(20*x^2 + sqrt(10)*(sqrt(5)*sqrt(2)*x -
5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) +
 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(20*x^2 - sqrt(10)*(sqrt(5)
*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(1/4) - 5*sqrt(2*sqrt(5) + 6)*(sq
rt(5) - 3)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(20*x^2 + sqr
t(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1/4) + 5*(sqrt(5)
 + 3)*sqrt(-2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*
log(20*x^2 - sqrt(10)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(1
/4) + 5*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))
```

giac [A] time = 0.93, size = 239, normalized size = 0.53

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{5\sqrt{5}+5} + \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(5*sqrt(5) + 5) + 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) - 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
```

```
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) + 1/40*sqrt(5*sqrt(5) + 5)
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

maple [C] time = 0.01, size = 42, normalized size = 0.09

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^7+12\,\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 0.18, size = 459, normalized size = 1.02

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{3\,2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(-\sqrt{5}-3\right)^{1/4}7\mathrm{i}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(2\sqrt{2}\sqrt{-\sqrt{5}-3}+\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}x\left(\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{3\,2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(\sqrt{5}-3\right)^{1/4}7\mathrm{i}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(2\sqrt{2}\sqrt{\sqrt{5}-3}-\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(-
5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (3*2^(3/4)*5
^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/
2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(2*2^(1/2)*(- 5^(1/2) -
3)^(1/2) + 2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(-
5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 2^(1/2)*5^(1/2
)*(- 5^(1/2) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*
atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) -
2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^
(1/4))/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1
/2))))*(5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2)
- 3)^(1/4)*7i)/(2*(2*2^(1/2)*(5^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2)
 - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(2*2^(1/2)*(5
^(1/2) - 3)^(1/2) - 2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1
/4)*1i)/20
```

sympy [A] time = 1.48, size = 24, normalized size = 0.05

$$\mathrm{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(25600t^5+16t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(25600*_t**5 + 1
6*_t + x)))
```

3.11 $\int \frac{1+x^4}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {28, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{1+2x^4+x^8}\,dx &= \int \frac{1}{1+x^4}\,dx\\ &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}\\ &= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int\right.}{}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.01, size = 95, normalized size = 1.12

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

giac [A] time = 0.39, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.00, size = 58, normalized size = 0.68

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{4}+\frac{\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*2^(1/2)*arctan(2^(1/2)*x-1)+1/8*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/
2)*x+1))+1/4*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 1.59, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 1.56, size = 33, normalized size = 0.39

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

sympy [A] time = 0.15, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.12 $\int \frac{1+x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}\right.$$

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 + ArcTan[(
1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/
8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqr
t[3]*x + x^2]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
```

```
q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} \\
&= \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{8}\int \\
&= -\frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x\right.
\end{aligned}$$

Mathematica [C] time = 0.17, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(-6\log\left(x^2-x+1\right)+6\log\left(x^2+x+1\right)+4i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-4i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+4\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x + x^2] +
6*Log[1 + x + x^2])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.46, size = 211, normalized size = 1.51

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(\sqrt{6}\sqrt{2}x+2x^2+2\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-\sqrt{6}\sqrt{2}x+2x^2+2\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*s
qrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6
)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(
3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sqrt(2)*l
```

```
og(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(
2)*x + 2*x^2 + 2) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x +
 1)
```

giac [A] time = 0.42, size = 108, normalized size = 0.77

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^2
 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3)) +
 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.02, size = 109, normalized size = 0.78

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\ln\left(x^2-\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{24}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)+1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/24*3^(1/2)*ln(x^
2-3^(1/2)*x+1)+1/4*arctan(2*x-3^(1/2))+1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/4
*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1) -
 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.14, size = 95, normalized size = 0.68

$$\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) + atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) + atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

sympy [C] time = 0.70, size = 190, normalized size = 1.36

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}+9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1+9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}+9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1+9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(9216t^5+8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 + 9216*(-1/8 - sqrt(3)*I/24)*
*5) + (-1/8 + sqrt(3)*I/24)*log(x - 1 + 9216*(-1/8 + sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (1/8 - sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 + 9216*(1/8 - sqrt
(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x + 1 + 9216*(1/8 + sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(92
16*_t**5 + 8*_t + x)))
```

3.13 $\int \frac{1+x^4}{1+x^8} \, dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2-\sqrt{2}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2+\sqrt{2}}}$$

Rubi [A] time = 0.25, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1413, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2-\sqrt{2}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^8),x]
```

```
[Out] -(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]
)/4 - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt
[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 +
 Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqr
t[2 - Sqrt[2]]])/4 - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]
]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - Sqr
t[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[2]]*x
 + x^2]/(8*Sqrt[2 + Sqrt[2]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1413

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[2*d*e, 2]}, Dist[e^2/(2*c), Int[1/(d + q*x^(n/2) + e*x^n), x], x] +
Dist[e^2/(2*c), Int[1/(d - q*x^(n/2) + e*x^n), x], x]] /; FreeQ[{a, c, d, e
}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{1+x^4}{1+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{2}x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2}x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2}}-x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}+x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}}$$

$$= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx +$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2}}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2-\sqrt{2}}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right)}{8\sqrt{2+\sqrt{2}}} + \frac{\log(}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} - \frac{\log\left(1\right.}{}$$

Mathematica [A] time = 0.19, size = 258, normalized size = 0.74

$$\frac{1}{8}\left(-\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\left(\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x-\cos\left(\frac{\pi}{8}\right)\right)\right)+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x+\cos\left(\frac{\pi}{8}\right)\right)\right)+2\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)+2\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[x*S
ec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]
]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2*x*Cos[Pi/8]]*(-Cos[Pi/8] + Sin[
Pi/8]) + 2*ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + 2*Ar
cTan[(x + Cos[Pi/8])*Csc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x
*Sin[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(Cos[Pi/
8] + Sin[Pi/8]))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 + x^8), x]
```

fricas [B] time = 1.34, size = 991, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x
*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sq
rt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sqrt(sqrt(
2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) +
2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*(sqrt(sqrt(2) + 2) +
 sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) -
 sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*ar
ctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2)
+ 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2)
 + 2)*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2
) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) + sqrt(-
sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*sqrt(2)*sqrt(
sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(
sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) +
sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*sqrt(2)
*sqrt(sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x
*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) +
 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/32*
sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*
sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) -
 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) +
 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*lo
g(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2)
+ 1) + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt(sqrt(
2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 - x*sq
rt(sqrt(2) + 2) + 1) + 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^
2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2
))*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

giac [A] time = 0.88, size = 247, normalized size = 0.71

$$\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) + 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2
) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt
(-sqrt(2) + 2)) + 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) +
 1) - 1/16*sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*s
qrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2*sqrt(2
) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.01, size = 27, normalized size = 0.08

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^8+1),x)

[Out] 1/8*sum((_R^4+1)/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^8+1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 + 1), x)

mupad [B] time = 2.28, size = 311, normalized size = 0.90

$$-\ln\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)+256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)+\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}+\frac{x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)-\frac{\operatorname{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\frac{\operatorname{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{2}-\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{4}+\frac{3}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 1)/(x^8 + 1),x)

[Out] atan((x*(2^(1/2) - 2)^(1/2)*1i)/2 + (x*(2^(1/2) + 2)^(1/2)*1i)/2 + (2^(1/2) *x*(2^(1/2) - 2)^(1/2)*1i)/2 - (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*((2^(1 /2)*(2^(1/2) - 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log(((- 2*2^(1/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(- 2*2^(1/2) - 4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) + 256)*((- 2*2^(1/2) - 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16) - (atan(x*(2^(1/2) + 2)^(3/2)*(1 - 1i/2) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 - 1i) - 2)*(2^(1/2) + 2)^(1/2)*1i)/8 + (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1 /2)*x*(2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2^(1/2) + 2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1/2 + 1i) + 2^(1/2)*(2^(1/2) + 2)^(3/2)*(1/4 + 3i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1/2) + 2)^(1 /2)/16)*1i

sympy [A] time = 2.78, size = 19, normalized size = 0.05

$$\operatorname{RootSum}\left(1048576t^8+1,\left(t\mapsto t\log\left(4096t^5+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)/(x**8+1),x)

[Out] RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 + 4*_t + x)))

3.14 $\int \frac{1+x^4}{1-x^4+x^8} \, dx$

Optimal. Leaf size=331

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{2+\sqrt{3}}}$$

Rubi [A] time = 0.23, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1419, 1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{2+\sqrt{3}}}-\frac{1}{4}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])/4
- (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4
 + (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/
4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])
/4 - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + Sqr
t[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]*x
 + x^2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[
2 + Sqrt[3]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{3}x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{3}x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{3}}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{3}}}$$

$$= \frac{1}{8}\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2} \cdots$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{2-\sqrt{3}}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{2-\sqrt{3}}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{8\sqrt{2+\sqrt{3}}} + \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{2+\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{2-\sqrt{3}}} - \cdots$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1^
7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.32, size = 377, normalized size = 1.14

$$-\frac{1}{8}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)\log\left(2x^2+2x\sqrt{\sqrt{3}+2}+2\right)+\frac{1}{8}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)\log\left(2x^2-2x\sqrt{\sqrt{3}+2}+2\right)+\frac{1}{16}\left(\sqrt{3}+2\right)\sqrt{-4\sqrt{3}+8}\log\left(2x^2+x\sqrt{-4\sqrt{3}+8}+2\right)-\frac{1}{16}\left(\sqrt{3}+2\right)\sqrt{-4\sqrt{3}+8}\log\left(2x^2-x\sqrt{-4\sqrt{3}+8}+2\right)-\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(\sqrt{2}\sqrt{2x^2+2x\sqrt{\sqrt{3}+2}+2}\sqrt{\sqrt{3}+2}-2x\sqrt{\sqrt{3}+2}-\sqrt{3}-2\right)-\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(\sqrt{2}\sqrt{2x^2-2x\sqrt{\sqrt{3}+2}+2}\sqrt{\sqrt{3}+2}-2x\sqrt{\sqrt{3}+2}+\sqrt{3}+2\right)-\frac{1}{4}\sqrt{-4\sqrt{3}+8}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{2x^2+x\sqrt{-4\sqrt{3}+8}+2}\sqrt{-4\sqrt{3}+8}-x\sqrt{-4\sqrt{3}+8}+\sqrt{3}-2\right)-\frac{1}{4}\sqrt{-4\sqrt{3}+8}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{2x^2-x\sqrt{-4\sqrt{3}+8}+2}\sqrt{-4\sqrt{3}+8}-x\sqrt{-4\sqrt{3}+8}-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(3) + 2)*(sqrt(3) - 2)*log(2*x^2 + 2*x*sqrt(sqrt(3) + 2) + 2)
 + 1/8*sqrt(sqrt(3) + 2)*(sqrt(3) - 2)*log(2*x^2 - 2*x*sqrt(sqrt(3) + 2) +
2) + 1/16*(sqrt(3) + 2)*sqrt(-4*sqrt(3) + 8)*log(2*x^2 + x*sqrt(-4*sqrt(3)
+ 8) + 2) - 1/16*(sqrt(3) + 2)*sqrt(-4*sqrt(3) + 8)*log(2*x^2 - x*sqrt(-4*s
qrt(3) + 8) + 2) - 1/2*sqrt(sqrt(3) + 2)*arctan(sqrt(2)*sqrt(2*x^2 + 2*x*sq
rt(sqrt(3) + 2) + 2)*sqrt(sqrt(3) + 2) - 2*x*sqrt(sqrt(3) + 2) - sqrt(3) -
2) - 1/2*sqrt(sqrt(3) + 2)*arctan(sqrt(2)*sqrt(2*x^2 - 2*x*sqrt(sqrt(3) + 2
) + 2)*sqrt(sqrt(3) + 2) - 2*x*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 1/4*sqrt(
-4*sqrt(3) + 8)*arctan(1/2*sqrt(2)*sqrt(2*x^2 + x*sqrt(-4*sqrt(3) + 8) + 2)
*sqrt(-4*sqrt(3) + 8) - x*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) - 1/4*sqrt(-4
*sqrt(3) + 8)*arctan(1/2*sqrt(2)*sqrt(2*x^2 - x*sqrt(-4*sqrt(3) + 8) + 2)*s
qrt(-4*sqrt(3) + 8) - x*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

giac [A] time = 0.50, size = 245, normalized size = 0.74

$$\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(6) - sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2)
)) + 1/8*(sqrt(6) - sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sq
rt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6)
 - sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sq
rt(6) - sqrt(2))) + 1/16*(sqrt(6) - sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/16*(sqrt(6) - sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) +
 1) + 1/16*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1
/16*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 42, normalized size = 0.13

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.22, size = 145, normalized size = 0.44

$$-\mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27-27\mathrm{i})}{27\sqrt{3}-81\mathrm{i}}\right)\left(\sqrt{2}\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right)-\mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27+27\mathrm{i})}{27\sqrt{3}-81\mathrm{i}}\right)\left(\sqrt{2}\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right)-\mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27-27\mathrm{i})}{27\sqrt{3}+81\mathrm{i}}\right)\left(\sqrt{2}\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)-\mathrm{atan}\left(\frac{\sqrt{6}\,x\,(27+27\mathrm{i})}{27\sqrt{3}+81\mathrm{i}}\right)\left(\sqrt{2}\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{6}\,\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((6^(1/2)*x*(27 - 27i))/(27*3^(1/2) - 81i))*(2^(1/2)*(1/8 + 1i/8) - 6
^(1/2)*(1/8 - 1i/8)) - atan((6^(1/2)*x*(27 + 27i))/(27*3^(1/2) - 81i))*(2^(
1/2)*(1/8 - 1i/8) + 6^(1/2)*(1/8 + 1i/8)) - atan((6^(1/2)*x*(27 - 27i))/(27
*3^(1/2) + 81i))*(2^(1/2)*(1/8 + 1i/8) + 6^(1/2)*(1/8 - 1i/8)) - atan((6^(1
/2)*x*(27 + 27i))/(27*3^(1/2) + 81i))*(2^(1/2)*(1/8 - 1i/8) - 6^(1/2)*(1/8
+ 1i/8))
```

sympy [**A**] time = 3.10, size = 20, normalized size = 0.06

$$\operatorname{RootSum}\left(65536t^8 - 256t^4 + 1, \left(t \mapsto t\log\left(1024t^5 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-x**4+1),x)
```

```
[Out] RootSum(65536*_t**8 - 256*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + x)))
```

3.15 $\int \frac{1+x^4}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=27

$$\frac{x}{2(1-x^4)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {28, 385, 212, 206, 203}

$$\frac{x}{2(1-x^4)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 - x^4)) + ArcTan[x]/4 + ArcTanh[x]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^4}{1-2x^4+x^8}\,dx &= \int \frac{1+x^4}{\left(-1+x^4\right)^2}\,dx \\
&= \frac{x}{2\left(1-x^4\right)} - \frac{1}{2}\int \frac{1}{-1+x^4}\,dx \\
&= \frac{x}{2\left(1-x^4\right)} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{2\left(1-x^4\right)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{8}\left(-\frac{4x}{x^4-1} - \log(1-x) + \log(x+1) + 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x)/(-1 + x^4) + 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 - 2*x^4 + x^8), x]
```

fricas [B] time = 1.49, size = 43, normalized size = 1.59

$$\frac{2\left(x^4-1\right)\arctan(x) + \left(x^4-1\right)\log\left(x+1\right) - \left(x^4-1\right)\log\left(x-1\right) - 4\,x}{8\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1) -
4*x)/(x^4 - 1)
```

giac [A] time = 0.52, size = 29, normalized size = 1.07

$$-\frac{x}{2\left(x^4-1\right)} + \frac{1}{4}\arctan(x) + \frac{1}{8}\log\left(|x+1|\right) - \frac{1}{8}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1)
)
```

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{1}{8(x+1)}-\frac{1}{8(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(x+1)+1/8*ln(x+1)+1/4/(x^2+1)*x+1/4*arctan(x)-1/8/(x-1)-1/8*ln(x-1)
```

maxima [A] time = 1.33, size = 27, normalized size = 1.00

$$-\frac{x}{2\left(x^4-1\right)}+\frac{1}{4}\arctan(x)+\frac{1}{8}\log(x+1)-\frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

mupad [B] time = 0.05, size = 21, normalized size = 0.78

$$\frac{\operatorname{atan}(x)}{4}+\frac{\operatorname{atanh}(x)}{4}-\frac{x}{2\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atan(x)/4 + atanh(x)/4 - x/(2*(x^4 - 1))
```

sympy [A] time = 0.15, size = 26, normalized size = 0.96

$$-\frac{x}{2x^4-2}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\frac{\operatorname{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] -x/(2*x**4 - 2) - log(x - 1)/8 + log(x + 1)/8 + atan(x)/4
```

3.16 $\int \frac{1+x^4}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=131

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
+ Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sq
rt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5]
)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{1-3x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{5}\,x^2+x^4}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{5}\,x^2+x^4}\,dx \\ &= \frac{1}{2}\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{2}\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2} \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(-1+\sqrt{5}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 131, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[Sqrt[2/(1
+ Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sq
rt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5]
)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.57, size = 247, normalized size = 1.89

$$-\frac{1}{2}\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(-\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{5}+1}+\frac{1}{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}\right)+\frac{1}{2}\sqrt{2}\sqrt{\sqrt{5}-1}\arctan\left(-\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{5}-1}+\frac{1}{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-1}\right)+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}+1}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}+4x\right)-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(sqrt(5) + 1)*arctan(-1/2*sqrt(2)*x*sqrt(sqrt(5) + 1) + 1/
2*sqrt(2*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1)) + 1/2*sqrt(2)*sqrt(sqrt(5) -
 1)*arctan(-1/2*sqrt(2)*x*sqrt(sqrt(5) - 1) + 1/2*sqrt(2*x^2 + sqrt(5) + 1)
*sqrt(sqrt(5) - 1)) + 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log((sqrt(5)*sqrt(2) -
sqrt(2))*sqrt(sqrt(5) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5) + 1)*log(-(sqr
t(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) + 4*x) - 1/8*sqrt(2)*sqrt(sqrt(5)
 - 1)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*x) + 1/8*sqrt(2
)*sqrt(sqrt(5) - 1)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*
x)
```

giac [A] time = 0.96, size = 147, normalized size = 1.12

$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{8}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{8}\sqrt{2\sqrt{5}+2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2*sqrt(5) - 2)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*sqrt(2*sqr
t(5) + 2)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/8*sqrt(2*sqrt(5) - 2)*log(a
bs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt
(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x + sqrt(1/2*sqrt(5
) - 1/2))) - 1/8*sqrt(2*sqrt(5) + 2)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

maple [A] time = 0.04, size = 96, normalized size = 0.73

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-3*x^4+1),x)
```

```
[Out] -1/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/(-2+2*5^(1/2))^(1
/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/(-2+2*5^(1/2))^(1/2)*arctanh(2/(-2+2
*5^(1/2))^(1/2)*x)-1/(2+2*5^(1/2))^(1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 3*x^4 + 1), x)
```

mupad [B] time = 0.20, size = 269, normalized size = 2.05

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}-1}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}+1}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{\sqrt{5}+1}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{5}}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}-\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{1-\sqrt{5}}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}-1875\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}\,1875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}+\frac{\sqrt{2}\,\sqrt{5}\,x\,\sqrt{-\sqrt{5}-1}\,875\mathrm{i}}{2\,\left(875\,\sqrt{5}+1875\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(1 - 5^(1/2))^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)
) - (2^(1/2)*5^(1/2)*x*(1 - 5^(1/2))^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*
(1 - 5^(1/2))^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1)^(1/2)*18
75i)/(2*(875*5^(1/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(5^(1/2) + 1)^(1/2)*875i
)/(2*(875*5^(1/2) + 1875)))*(5^(1/2) + 1)^(1/2)*1i)/4 - (2^(1/2)*atan((2^(1
/2)*x*(5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1/2) - 1875)) - (2^(1/2)*5^(1/2
)*x*(5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/2) - 1875)))*(5^(1/2) - 1)^(1/2)
*1i)/4 + (2^(1/2)*atan((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1875i)/(2*(875*5^(1
/2) + 1875)) + (2^(1/2)*5^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*875i)/(2*(875*5^(1/
2) + 1875)))*(- 5^(1/2) - 1)^(1/2)*1i)/4
```

sympy [A] time = 1.19, size = 49, normalized size = 0.37

$$\operatorname{RootSum}\left(256t^4-16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right)+\operatorname{RootSum}\left(256t^4+16t^2-1,\left(t\mapsto t\log\left(1024t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(256*_t**4 - 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t + x))
) + RootSum(256*_t**4 + 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 - 8*_t +
 x)))
```

3.17 $\int \frac{1+x^4}{1-4x^4+x^8} \, dx$

Optimal. Leaf size=157

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

Rubi [A] time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[3]]) - Arc
Tan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 + Sqrt[3]]) + ArcTanh[
(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[3]]) - ArcTanh[(2
^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 + Sqrt[3]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{6}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{6}\,x^2+x^4}\,dx$$

$$= \frac{\int \frac{1}{-\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}} + \frac{\int \frac{1}{\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{2}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{-1+\sqrt{3}}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.34

$$\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{\#1^7-2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 4*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1^3 + #
1^7) & ]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1-4x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 - 4*x^4 + x^8), x]
```

fricas [B] time = 1.26, size = 331, normalized size = 2.11

$$\frac{1}{2}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{2}\sqrt{x^2+\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}}\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}-\frac{1}{2}\left(\sqrt{3}\sqrt{2}x+\sqrt{2}x\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}\right)-\frac{1}{2}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{2}\left(\sqrt{x^2-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)}\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{3}+2}-\left(\sqrt{3}\sqrt{2}x-\sqrt{2}x\right)\sqrt{\sqrt{3}+2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)-\frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)+\frac{1}{8}\sqrt{2}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(-sqrt(3) + 2)^(1/4)*arctan(1/2*sqrt(x^2 + (sqrt(3) + 2)*sqrt(-
sqrt(3) + 2))*(sqrt(3)*sqrt(2) + sqrt(2))*(-sqrt(3) + 2)^(3/4) - 1/2*(sqrt(
3)*sqrt(2)*x + sqrt(2)*x)*(-sqrt(3) + 2)^(3/4)) - 1/2*sqrt(2)*(sqrt(3) + 2)
^(1/4)*arctan(1/2*(sqrt(x^2 - sqrt(sqrt(3) + 2)*(sqrt(3) - 2))*(sqrt(3)*sqr
t(2) - sqrt(2))*sqrt(sqrt(3) + 2) - (sqrt(3)*sqrt(2)*x - sqrt(2)*x)*sqrt(sq
rt(3) + 2))*(sqrt(3) + 2)^(1/4)) + 1/8*sqrt(2)*(sqrt(3) + 2)^(1/4)*log((sqr
t(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*x) - 1/8*sqrt(2)*(sqrt(3) +
 2)^(1/4)*log(-(sqrt(3)*sqrt(2) - sqrt(2))*(sqrt(3) + 2)^(1/4) + 2*x) - 1/8
*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log((sqrt(3)*sqrt(2) + sqrt(2))*(-sqrt(3) + 2
)^(1/4) + 2*x) + 1/8*sqrt(2)*(-sqrt(3) + 2)^(1/4)*log(-(sqrt(3)*sqrt(2) + s
qrt(2))*(-sqrt(3) + 2)^(1/4) + 2*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 40, normalized size = 0.25

$$\frac{\left(\mathrm{RootOf}\left(\_Z^{8}-4\_Z^{4}+1\right)^{4}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-4\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-4\_Z^{4}+1\right)^{7}-16\,\mathrm{RootOf}\left(\_Z^{8}-4\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-4*x^4+1),x)
```

```
[Out] 1/8*sum((_R^4+1)/(_R^7-2*_R^3)*ln(-_R+x),_R=RootOf(_Z^8-4*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8-4\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

mupad [B] time = 1.72, size = 399, normalized size = 2.54

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\,\sqrt{2}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{3024\,\sqrt{2}\,\sqrt{3}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}\right)\,{\left(\sqrt{3}+2\right)}^{1/4}}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}\,5184{}\mathrm{i}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}-\frac{\sqrt{2}\,\sqrt{3}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}\,3024{}\mathrm{i}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}\right)\,{\left(2-\sqrt{3}\right)}^{1/4}\,1{}\mathrm{i}}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{5184\,\sqrt{2}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}-\frac{3024\,\sqrt{2}\,\sqrt{3}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}}{2160\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-3888\,\sqrt{2-\sqrt{3}}}\right)\,{\left(2-\sqrt{3}\right)}^{1/4}}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}\,5184{}\mathrm{i}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{\sqrt{2}\,\sqrt{3}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}\,3024{}\mathrm{i}}{3888\,\sqrt{\sqrt{3}+2}+2160\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}\right)\,{\left(\sqrt{3}+2\right)}^{1/4}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(2 - 3^(1/2))^(1/4)*5184i)/(2160*3^(1/2)*(2 - 3^(1
/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (2^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(
1/4)*3024i)/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)))*
(2 - 3^(1/2))^(1/4)*1i)/4 - (2^(1/2)*atan((5184*2^(1/2)*x*(2 - 3^(1/2))^(1/
4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3888*(2 - 3^(1/2))^(1/2)) - (3024*2
^(1/2)*3^(1/2)*x*(2 - 3^(1/2))^(1/4))/(2160*3^(1/2)*(2 - 3^(1/2))^(1/2) - 3
888*(2 - 3^(1/2))^(1/2)))*(2 - 3^(1/2))^(1/4))/4 + (2^(1/2)*atan((5184*2^(1
/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2
) + 2)^(1/2)) + (3024*2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3888*(3^(1/2)
 + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(3^(1/2) + 2)^(1/4))/4 - (
2^(1/2)*atan((2^(1/2)*x*(3^(1/2) + 2)^(1/4)*5184i)/(3888*(3^(1/2) + 2)^(1/2
) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (2^(1/2)*3^(1/2)*x*(3^(1/2) + 2)^(1
/4)*3024i)/(3888*(3^(1/2) + 2)^(1/2) + 2160*3^(1/2)*(3^(1/2) + 2)^(1/2)))*(
3^(1/2) + 2)^(1/4)*1i)/4
```

sympy [A] time = 0.19, size = 24, normalized size = 0.15

$$\mathrm{RootSum}\left(1048576t^{8}-4096t^{4}+1,\left(t\mapsto t\log\left(4096t^{5}-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] RootSum(1048576*_t**8 - 4096*_t**4 + 1, Lambda(_t, _t*log(4096*_t**5 - 12*_
t + x)))
```

3.18 $\int \frac{1+x^4}{1-5x^4+x^8} \, dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7])] - ArcTa
n[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt
[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7])] - ArcTanh[Sqrt[2/(
Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt[7])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{7}x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{7}x^2+x^4}\,dx$$

$$= \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}} + \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(-\sqrt{3}+\sqrt{7}\right)}} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.32

$$\frac{1}{4}\text{RootSum}\left[\#1^8-5\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7-5\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 5*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1^3 + 2
*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1-5x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 - 5*x^4 + x^8), x]
```

fricas [B] time = 1.65, size = 574, normalized size = 3.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*arctan(1/48*(sqrt(7)*s
qrt(6)*sqrt(3)*sqrt(2) + 3*sqrt(6)*sqrt(2))*sqrt(4*x^2 + (sqrt(7)*sqrt(3)*s
qrt(2) + 5*sqrt(2))*sqrt(-sqrt(7)*sqrt(3) + 5))*sqrt(-sqrt(7)*sqrt(3) + 5)*
sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) - 1/24*(sqrt(7)*sqrt(6)*sqrt(3)*sq
rt(2)*x + 3*sqrt(6)*sqrt(2)*x)*sqrt(-sqrt(7)*sqrt(3) + 5)*sqrt(sqrt(2)*sqrt
(-sqrt(7)*sqrt(3) + 5))) - 1/6*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) +
5))*arctan(1/48*((sqrt(7)*sqrt(6)*sqrt(3)*sqrt(2) - 3*sqrt(6)*sqrt(2))*sqrt
(4*x^2 - (sqrt(7)*sqrt(3)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(7)*sqrt(3) + 5))*s
qrt(sqrt(7)*sqrt(3) + 5) - 2*(sqrt(7)*sqrt(6)*sqrt(3)*sqrt(2)*x - 3*sqrt(6)
*sqrt(2)*x)*sqrt(sqrt(7)*sqrt(3) + 5))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) +
5))) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sq
rt(6)*sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x)
- 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6
)*sqrt(3) - 3*sqrt(6))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 12*x) - 1/
24*sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log((sqrt(7)*sqrt(6)*sq
rt(3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x) + 1/24*
```

```
sqrt(6)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*log(-(sqrt(7)*sqrt(6)*sqrt
(3) + 3*sqrt(6))*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) + 12*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 42, normalized size = 0.25

$$\frac{\left(\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)^{4}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)^{7}-20\,\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-5*x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-5*_R^3)*ln(-_R+x),_R=RootOf(_Z^8-5*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{x^{8}-5\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

mupad [B] time = 1.76, size = 483, normalized size = 2.82

$$\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right){\left(5-\sqrt{21}\right)}^{1/4}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}\,12005{}\mathrm{i}}{2\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}\,7889{}\mathrm{i}}{6\left(4802\sqrt{2}\sqrt{5-\sqrt{21}}-1029\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right){\left(5-\sqrt{21}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{12005\,2^{3/4}\sqrt{3}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{7889\,2^{3/4}\sqrt{3}\sqrt{21}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right){\left(\sqrt{21}+5\right)}^{1/4}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{3}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}\,12005{}\mathrm{i}}{2\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{3}\sqrt{21}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}\,7889{}\mathrm{i}}{6\left(4802\sqrt{2}\sqrt{\sqrt{21}+5}+1029\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right){\left(\sqrt{21}+5\right)}^{1/4}\,1{}\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 5*x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((12005*2^(3/4)*3^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(48
02*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2
))) - (7889*2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(6*(4802*2^(1/
2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5
- 21^(1/2))^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(5 - 21^(1
/2))^(1/4)*12005i)/(2*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^
(1/2)*(5 - 21^(1/2))^(1/2))) - (2^(3/4)*3^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(
1/4)*7889i)/(6*(4802*2^(1/2)*(5 - 21^(1/2))^(1/2) - 1029*2^(1/2)*21^(1/2)*(
5 - 21^(1/2))^(1/2))))*(5 - 21^(1/2))^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan(
(12005*2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(4802*2^(1/2)*(21^(1/2) +
 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (7889*2^(3/4)*3^
(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4))/(6*(4802*2^(1/2)*(21^(1/2) + 5)^(1/2
) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(3/4)*3^(1/2)*x*(21^(1/2) + 5)^(1/4)*12005i)/(2*
(4802*2^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(
1/2))) + (2^(3/4)*3^(1/2)*21^(1/2)*x*(21^(1/2) + 5)^(1/4)*7889i)/(6*(4802*2
```

```
^(1/2)*(21^(1/2) + 5)^(1/2) + 1029*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2)))
*(21^(1/2) + 5)^(1/4)*1i)/12
```

sympy [A] time = 0.19, size = 24, normalized size = 0.14

$$\operatorname{RootSum}\left(5308416t^8 - 11520t^4 + 1, \left(t \mapsto t \log\left(9216t^5 - 16t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 11520*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 16*
_t + x)))
```

3.19 $\int \frac{1+x^4}{1-6x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1419, 1093, 203, 207}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sqrt[1 + Sqr
t[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + S
qrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{1-6x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-2\sqrt{2}\,x^2+x^4}\,dx + \frac{1}{2}\int \frac{1}{1+2\sqrt{2}\,x^2+x^4}\,dx \\ &= \frac{1}{4}\int \frac{1}{-1-\sqrt{2}+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-\sqrt{2}+x^2}\,dx + \frac{1}{4}\int \frac{1}{-1+\sqrt{2}+x^2}\,dx - \frac{1}{4}\int \frac{1}{1+\sqrt{2}+x^2} \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 111, normalized size = 0.95

$$\frac{1}{4}\left(\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[2]]*ArcTan
[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] - S
qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{1-6x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^4)/(1 - 6*x^4 + x^8), x]
```

fricas [B] time = 1.23, size = 181, normalized size = 1.55

$$-\frac{1}{2}\sqrt{\sqrt{2}+1}\arctan\left(-x\sqrt{\sqrt{2}+1}+\sqrt{x^2+\sqrt{2}-1}\sqrt{\sqrt{2}+1}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\arctan\left(-x\sqrt{\sqrt{2}-1}+\sqrt{x^2+\sqrt{2}+1}\sqrt{\sqrt{2}-1}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(sqrt(2) + 1)*arctan(-x*sqrt(sqrt(2) + 1) + sqrt(x^2 + sqrt(2) - 1
)*sqrt(sqrt(2) + 1)) + 1/2*sqrt(sqrt(2) - 1)*arctan(-x*sqrt(sqrt(2) - 1) +
sqrt(x^2 + sqrt(2) + 1)*sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*log((sqr
t(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(sqrt(2) - 1)*log(-(sqrt(2) + 1)
*sqrt(sqrt(2) - 1) + x) + 1/8*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt
(2) - 1) + x) - 1/8*sqrt(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1)
+ x)
```

giac [A] time = 0.91, size = 123, normalized size = 1.05

$$-\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(sqrt(2) - 1)*arctan(x/sqrt(sqrt(2) + 1)) + 1/4*sqrt(sqrt(2) + 1)*
arctan(x/sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*log(abs(x + sqrt(sqrt(2
) + 1))) + 1/8*sqrt(sqrt(2) - 1)*log(abs(x - sqrt(sqrt(2) + 1))) + 1/8*sqrt
(sqrt(2) + 1)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/8*sqrt(sqrt(2) + 1)*log(a
bs(x - sqrt(sqrt(2) - 1)))
```

maple [A] time = 0.06, size = 78, normalized size = 0.67

$$-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^8-6*x^4+1),x)
```

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(2^(1/2)-1)
^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^8-6\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 6*x^4 + 1), x)
```

mupad [B] time = 0.19, size = 233, normalized size = 1.99

$$-\frac{\operatorname{atan}\left(\frac{x\,\sqrt{\sqrt{2}-1}\,49152{}\mathrm{i}}{34816\,\sqrt{2}-49152}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}-1}\,34816{}\mathrm{i}}{34816\,\sqrt{2}-49152}\right)\,\sqrt{\sqrt{2}-1}\,1{}\mathrm{i}}{4}-\frac{\operatorname{atan}\left(\frac{x\,\sqrt{\sqrt{2}+1}\,49152{}\mathrm{i}}{34816\,\sqrt{2}+49152}+\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+1}\,34816{}\mathrm{i}}{34816\,\sqrt{2}+49152}\right)\,\sqrt{\sqrt{2}+1}\,1{}\mathrm{i}}{4}+\frac{\operatorname{atan}\left(\frac{x\,\sqrt{1-\sqrt{2}}\,49152{}\mathrm{i}}{34816\,\sqrt{2}-49152}-\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{2}}\,34816{}\mathrm{i}}{34816\,\sqrt{2}-49152}\right)\,\sqrt{1-\sqrt{2}}\,1{}\mathrm{i}}{4}+\frac{\operatorname{atan}\left(\frac{x\,\sqrt{-\sqrt{2}-1}\,49152{}\mathrm{i}}{34816\,\sqrt{2}+49152}+\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{2}-1}\,34816{}\mathrm{i}}{34816\,\sqrt{2}+49152}\right)\,\sqrt{-\sqrt{2}-1}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^8 - 6*x^4 + 1),x)
```

```
[Out] (atan((x*(1 - 2^(1/2))^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2^(1/2)*x*(
1 - 2^(1/2))^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(1 - 2^(1/2))^(1/2)*1i)
/4 - (atan((x*(2^(1/2) + 1)^(1/2)*49152i)/(34816*2^(1/2) + 49152) + (2^(1/2
)*x*(2^(1/2) + 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(2^(1/2) + 1)^(1/2
)*1i)/4 - (atan((x*(2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) - 49152) - (2
^(1/2)*x*(2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) - 49152))*(2^(1/2) - 1)
^(1/2)*1i)/4 + (atan((x*(- 2^(1/2) - 1)^(1/2)*49152i)/(34816*2^(1/2) + 4915
2) + (2^(1/2)*x*(- 2^(1/2) - 1)^(1/2)*34816i)/(34816*2^(1/2) + 49152))*(- 2
^(1/2) - 1)^(1/2)*1i)/4
```

sympy [A] time = 1.16, size = 49, normalized size = 0.42

$$\operatorname{RootSum}\left(4096t^4-128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)+\operatorname{RootSum}\left(4096t^4+128t^2-1,\left(t\mapsto t\log\left(16384t^5-20t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] RootSum(4096*_t**4 - 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 - 20*_t +
 x))) + RootSum(4096*_t**4 + 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5 -
 20*_t + x)))
```

3.20 $\int \frac{1-x^4}{1+bx^4+x^8} \, dx$

Optimal. Leaf size=511

$$\frac{\sqrt{2-\sqrt{2-b}}\,\log\left(-\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{2-\sqrt{2-b}}\,\log\left(\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{\sqrt{2-b}+2}\,\log(}{}$$

Rubi [A] time = 0.36, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{2-\sqrt{2-b}}\,\log\left(-\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{2-\sqrt{2-b}}\,\log\left(\sqrt{2-\sqrt{2-b}}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{\sqrt{2-b}+2}\,\log\left(-\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{2-b}}+\frac{\sqrt{\sqrt{2-b}+2}\,\log\left(\sqrt{\sqrt{2-b}+2}\,x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{b+2}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}}+\frac{\sqrt{b+2}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\,\sqrt{2-b}}+\frac{\sqrt{b+2}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}}-\frac{\sqrt{b+2}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\,\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]])/
(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqrt[
2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 - b]
) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]
])/(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sq
rt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 -
 b]) + (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2])/(8*Sq
rt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x^2])
/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 - Sqrt[2 + Sqrt[2 - b]]*x +
 x^2])/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 - b
]]*x + x^2])/(8*Sqrt[2 - b])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx = -\frac{\int \frac{\sqrt{2-b}+2x^2}{-1-\sqrt{2-b}\,x^2-x^4}\,dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x^2}{-1+\sqrt{2-b}\,x^2-x^4}\,dx}{2\sqrt{2-b}}$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}-\left(-2+\sqrt{2-b}\right)x}{1-\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}} + \frac{\int \frac{\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}+\left(-2+\sqrt{2-b}\right)x}{1+\sqrt{2-\sqrt{2-b}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2-b}}\,\sqrt{2-b}} + \frac{\int \frac{\sqrt{2+\sqrt{2-b}}\,\sqrt{2-b}-(2+\cdots}{1-\sqrt{2+\sqrt{2-b}}\,x+x\cdots}}{4\sqrt{2+\sqrt{2-b}}\,\sqrt{\cdots}}$$

$$= -\left(\frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1-\sqrt{2+\sqrt{2-b}}\,x+x^2}\,dx\right) - \frac{1}{8}\left(-1+\frac{2}{\sqrt{2-b}}\right)\int \frac{1}{1+\sqrt{2+\sqrt{\cdots}}}$$

$$= \frac{\sqrt{2-\sqrt{2-b}}\,\log\left(1-\sqrt{2-\sqrt{2-b}}\,x+x^2\right)}{8\sqrt{2-b}} - \frac{\sqrt{2-\sqrt{2-b}}\,\log\left(1+\sqrt{2-\sqrt{2-b}}\,x+\cdots\right.}{8\sqrt{2-b}}$$

$$= -\frac{\sqrt{2+\sqrt{2-b}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{2+\sqrt{2-b}}}\right)}{4\sqrt{2-b}} + \frac{\sqrt{2-\sqrt{2-b}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2-b}}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-b}} + \frac{\sqrt{2+\sqrt{2-\cdots}}}{\cdots}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.11

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+\#1^4 b+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7+\#1^3 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 + b*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(b*#1^
3 + 2*#1^7) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1+bx^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 + b*x^4 + x^8), x]
```

fricas [B] time = 1.35, size = 1443, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)
) - b)/(b^2 - 4*b + 4)))*arctan(1/2*sqrt(1/2)*(b^2 + (b^3 - 6*b^2 + 12*b -
8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/
2)*(b^2 + (b^3 - 6*b^2 + 12*b - 8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) -
 2*b)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^
2 - 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b
^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(
b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)) - 1/2*sqrt(1/2)*((b^3 - 6*b^
2 + 12*b - 8)*x*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + (b^2 - 4*b + 4)*x)
*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)
) - b)/(b^2 - 4*b + 4)))*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 +
12*b - 8)) - b)/(b^2 - 4*b + 4))) + sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*s
qrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*arctan(-1/2*(s
qrt(1/2)*(b^2 - (b^3 - 6*b^2 + 12*b - 8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b -
 8)) - 4*b + 4)*sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 - 6*b^2 + 12*b - 8)*sq
rt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - 2*b)*sqrt(-((b^2 - 4*b + 4)*sqrt((b
+ 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*sqrt(-((b^2 - 4*b + 4
)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)) + sqrt(1/2)*
((b^3 - 6*b^2 + 12*b - 8)*x*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - (b^2 -
 4*b + 4)*x)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8))
+ b)/(b^2 - 4*b + 4)))*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(
b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))) + 1/4*sqrt(sqrt(1/2)*sqrt(
-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b +
4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b +
2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b -
 8)) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4
)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4)))*log(-1/2*((
b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sqrt(sqrt(1/
2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2
- 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(
b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*log(1/2*((b^2 - 4*b + 4)*sq
rt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4
*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x)
+ 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b
 - 8)) - b)/(b^2 - 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 -
 6*b^2 + 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b +
 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.75Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.00, size = 44, normalized size = 0.09

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)^7+4\,\mathrm{RootOf}\left(\_Z^8+b\_Z^4+1\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+b*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8+_Z^4*b+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8+bx^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+b*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

mupad [B] time = 3.74, size = 5341, normalized size = 10.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(b*x^4 + x^8 + 1),x)
```

```
[Out] - atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 3
2*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2
) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b
- 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 2
62144) + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*
b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/
(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) -
x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*
b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i - ((-(4*b + (
(b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 1966
08*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) - x*(32768*b
 - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65
536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b
 - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) + x*(32*b + 48*b^2 +
24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24
*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b + ((b - 2)^5*(b + 2))^
(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b
 + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b -
 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4
+ 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) + x*(32768*b - 65536*b^2 - 3276
8*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b
- 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16
)))^(3/4) - 64*b^3 - 16*b^4 + 256) - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-
(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3
 + b^4 + 16)))^(1/4) + ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(
512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b + ((b - 2)^
5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(
1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6
 - 4096*b^7 + 262144) - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10
240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2)
```

```
- 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 1
6*b^4 + 256) + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b +
 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)))
*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*
b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b + ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b
 - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 1
6)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 40
96*b^6 - 4096*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480
*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2
))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i
- 256*b + 64*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(
-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^
3 + b^4 + 16)))^(1/4) - ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/
(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b +
2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(2
62144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096
*b^7 + 262144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*
b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*
b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*
b^3 + 16*b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b -
 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)
))^(1/4))/(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
- 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4
*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196
608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144
)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^
6 - 1024*b^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4
- 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2)
)^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i +
 ((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8
*b^3 + b^4 + 16)))^(1/4)*(((-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3
)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 -
196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144)*1i - x*(
32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b
^7 + 65536))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4 - 256)*1i
- x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b + ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i))*(-(4*b +
((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4
+ 16)))^(1/4) - atan((((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(5
12*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)^5
*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1
/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6
- 4096*b^7 + 262144) + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 102
40*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) -
 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16
*b^4 + 256) - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b +
2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i
 - ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b -
 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4
*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196
608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144
) - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 -
 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*
(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) + x*(32
*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 +
 b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*1i)/(((-(4*b - ((b -
```

```
2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16))
)^(1/4)*(256*b + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(2
4*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^
3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 262144) + x*(32768*b - 65
536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))
*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*
b^3 + b^4 + 16)))^(3/4) - 64*b^3 - 16*b^4 + 256) - x*(32*b + 48*b^2 + 24*b^
3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
- 32*b - 8*b^3 + b^4 + 16)))^(1/4) + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) -
4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(256*b + ((-(4
*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 +
 b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*
b^5 - 4096*b^6 - 4096*b^7 + 262144) - x*(32768*b - 65536*b^2 - 32768*b^3 +
20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(
b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4
) - 64*b^3 - 16*b^4 + 256) + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - (
(b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
 16)))^(1/4)))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b
^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*2i - 2*atan((((-(4*b - ((b - 2)^5*(b
+ 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*
(((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8
*b^3 + b^4 + 16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 +
49152*b^5 - 4096*b^6 - 4096*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 327
68*b^3 + 20480*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b
 - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 1
6)))^(3/4)*1i - 256*b + 64*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b
^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2
 - 32*b - 8*b^3 + b^4 + 16)))^(1/4) - ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) -
 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((
b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 +
16)))^(1/4)*(262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4
096*b^6 - 4096*b^7 + 262144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 2048
0*b^4 + 10240*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b +
2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i
 - 256*b + 64*b^3 + 16*b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*
(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b
^3 + b^4 + 16)))^(1/4))/(((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)
/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)^5*(b +
 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(
262144*b - 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 409
6*b^7 + 262144)*1i + x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240
*b^5 - 2048*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4
*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64
*b^3 + 16*b^4 - 256)*1i + x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b
- 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16
)))^(1/4)*1i + ((-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*
b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(((-(4*b - ((b - 2)^5*(b + 2))^(1/2)
 - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*(262144*b -
 196608*b^2 - 196608*b^3 + 49152*b^4 + 49152*b^5 - 4096*b^6 - 4096*b^7 + 26
2144)*1i - x*(32768*b - 65536*b^2 - 32768*b^3 + 20480*b^4 + 10240*b^5 - 204
8*b^6 - 1024*b^7 + 65536))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3
)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(3/4)*1i - 256*b + 64*b^3 + 16*
b^4 - 256)*1i - x*(32*b + 48*b^2 + 24*b^3 + 4*b^4))*(-(4*b - ((b - 2)^5*(b
+ 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b - 8*b^3 + b^4 + 16)))^(1/4)*
1i))*(-(4*b - ((b - 2)^5*(b + 2))^(1/2) - 4*b^2 + b^3)/(512*(24*b^2 - 32*b
- 8*b^3 + b^4 + 16)))^(1/4)
```

sympy [**A**] time = 3.63, size = 76, normalized size = 0.15

$$-\operatorname{RootSum}\left(t^{8}\left(65536b^{4}-524288b^{3}+1572864b^{2}-2097152b+1048576\right)+t^{4}\left(256b^{3}-1024b^{2}+1024b\right)+1,\left(t\mapsto t\log\left(1024t^{5}b^{2}-4096t^{5}b+4096t^{5}+4tb-4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] -RootSum(_t**8*(65536*b**4 - 524288*b**3 + 1572864*b**2 - 2097152*b + 10485
76) + _t**4*(256*b**3 - 1024*b**2 + 1024*b) + 1, Lambda(_t, _t*log(1024*_t*
*5*b**2 - 4096*_t**5*b + 4096*_t**5 + 4*_t*b - 4*_t + x)))
```

3.21 $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=411

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\right.}\right.}{4\,2^{3/4}}$$

Rubi [A] time = 0.32, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1420, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/
4)) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*
2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]
)/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(2*2^(3/4)) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*
(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqr
t[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((
3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1420

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && GtQ[b^2
 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx = \frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,d$$

$$= -\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}} + \frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}\right.}{4\,2^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt[4]{2\left(3+\sqrt{5}\right)}} -$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 + 3*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1+3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 + 3*x^4 + x^8), x]
```

fricas [B] time = 1.58, size = 894, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(5)*sqrt(2) - 3*sqrt(2))*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*
arctan(1/16*sqrt(4*x^2 - sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3) + 2*(sqrt(5)*x -
 x)*(2*sqrt(5) + 6)^(1/4))*(sqrt(5)*sqrt(2) - 2*sqrt(2))*(2*sqrt(5) + 6)^(5
/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2)*x - 2*sqrt(2)*x)*(2*sqrt(5) +
6)^(5/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(
5) + 6)*sqrt(sqrt(5) + 3)) + 1/16*(sqrt(5)*sqrt(2) - 3*sqrt(2))*(2*sqrt(5)
+ 6)^(3/4)*sqrt(sqrt(5) + 3)*arctan(1/16*sqrt(4*x^2 - sqrt(2*sqrt(5) + 6)*(
sqrt(5) - 3) - 2*(sqrt(5)*x - x)*(2*sqrt(5) + 6)^(1/4))*(sqrt(5)*sqrt(2) -
2*sqrt(2))*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2)*x
 - 2*sqrt(2)*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt
(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/16*(sqrt(5)*sqr
t(2) + 3*sqrt(2))*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/16*sqr
t(4*x^2 + (sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6) + 2*(sqrt(5)*x + x)*(-2*sqrt(5
) + 6)^(1/4))*(sqrt(5)*sqrt(2) + 2*sqrt(2))*sqrt(-sqrt(5) + 3)*(-2*sqrt(5)
+ 6)^(5/4) - 1/8*((sqrt(5)*sqrt(2)*x + 2*sqrt(2)*x)*(-2*sqrt(5) + 6)^(5/4)
+ (sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) +
 1/16*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/
4)*arctan(1/16*sqrt(4*x^2 + (sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6) - 2*(sqrt(5)
*x + x)*(-2*sqrt(5) + 6)^(1/4))*(sqrt(5)*sqrt(2) + 2*sqrt(2))*sqrt(-sqrt(5)
 + 3)*(-2*sqrt(5) + 6)^(5/4) - 1/8*((sqrt(5)*sqrt(2)*x + 2*sqrt(2)*x)*(-2*s
qrt(5) + 6)^(5/4) - (sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqr
t(-sqrt(5) + 3)) + 1/8*(2*sqrt(5) + 6)^(1/4)*log(4*x^2 - sqrt(2*sqrt(5) + 6
)*(sqrt(5) - 3) + 2*(sqrt(5)*x - x)*(2*sqrt(5) + 6)^(1/4)) - 1/8*(2*sqrt(5)
 + 6)^(1/4)*log(4*x^2 - sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3) - 2*(sqrt(5)*x -
x)*(2*sqrt(5) + 6)^(1/4)) - 1/8*(-2*sqrt(5) + 6)^(1/4)*log(4*x^2 + (sqrt(5)
 + 3)*sqrt(-2*sqrt(5) + 6) + 2*(sqrt(5)*x + x)*(-2*sqrt(5) + 6)^(1/4)) + 1/
8*(-2*sqrt(5) + 6)^(1/4)*log(4*x^2 + (sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6) - 2
*(sqrt(5)*x + x)*(-2*sqrt(5) + 6)^(1/4))
```

giac [A] time = 0.69, size = 223, normalized size = 0.54

$$\frac{1}{16}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{\sqrt{5}+1} - \frac{1}{16}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{\sqrt{5}-1} + \frac{1}{16}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{\sqrt{5}-1} - \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 2500x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}-1}\log\left(2500\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 2500x^2\right) + \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 1156x^2\right) - \frac{1}{8}\sqrt{\sqrt{5}+1}\log\left(1156\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 1156x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi
+ 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(sqrt(5) + 1) - 1/16*(pi + 4*arct
an(x*sqrt(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) + 1/16*(pi + 4*arctan(-x*sqr
t(sqrt(5) - 1) - 1))*sqrt(sqrt(5) - 1) - 1/8*sqrt(sqrt(5) - 1)*log(2500*(x
+ sqrt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) - 1)*log(2500*(x - sq
rt(sqrt(5) + 1))^2 + 2500*x^2) + 1/8*sqrt(sqrt(5) + 1)*log(1156*(x + sqrt(s
qrt(5) - 1))^2 + 1156*x^2) - 1/8*sqrt(sqrt(5) + 1)*log(1156*(x - sqrt(sqrt(
5) - 1))^2 + 1156*x^2)
```

maple [C] time = 0.01, size = 44, normalized size = 0.11

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{4}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)+x\right)}{8\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{7}+12\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(_Z^8+3*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{4}-1}{x^{8}+3\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

mupad [B] time = 1.68, size = 447, normalized size = 1.09

$$\frac{2^{3/4}\,\mathrm{atan}\left(\frac{1875\,2^{3/4}\,x\,(\sqrt{5}-3)^{1/4}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{875\,2^{3/4}\sqrt{5}\,x\,(\sqrt{5}-3)^{1/4}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)(\sqrt{5}-3)^{1/4}}{4}-\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,(\sqrt{5}-3)^{1/4}\,1875\,\mathrm{i}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,(\sqrt{5}-3)^{1/4}\,875\,\mathrm{i}}{2\left(625\sqrt{2}\sqrt{\sqrt{5}-3}-250\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}\right)}\right)(\sqrt{5}-3)^{1/4}\,1\mathrm{i}}{4}+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{1875\,2^{3/4}\,x\,(-\sqrt{5}-3)^{1/4}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{875\,2^{3/4}\sqrt{5}\,x\,(-\sqrt{5}-3)^{1/4}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)(-\sqrt{5}-3)^{1/4}}{4}-\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,(-\sqrt{5}-3)^{1/4}\,1875\,\mathrm{i}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,(-\sqrt{5}-3)^{1/4}\,875\,\mathrm{i}}{2\left(625\sqrt{2}\sqrt{-\sqrt{5}-3}+250\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)}\right)(-\sqrt{5}-3)^{1/4}\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*atan((1875*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2)
 - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (875*2^(3/4)*5^(1
/2)*x*(5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2
)*5^(1/2)*(5^(1/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^
(3/4)*x*(5^(1/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 25
0*2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^
(1/4)*875i)/(2*(625*2^(1/2)*(5^(1/2) - 3)^(1/2) - 250*2^(1/2)*5^(1/2)*(5^(1
/2) - 3)^(1/2))))*(5^(1/2) - 3)^(1/4)*1i)/4 + (2^(3/4)*atan((1875*2^(3/4)*x
*(- 5^(1/2) - 3)^(1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)
*5^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (875*2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(
1/4))/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2)*(- 5^(1/2
) - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4))/4 - (2^(3/4)*atan((2^(3/4)*x*(- 5^(1
/2) - 3)^(1/4)*1875i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5
^(1/2)*(- 5^(1/2) - 3)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*8
75i)/(2*(625*2^(1/2)*(- 5^(1/2) - 3)^(1/2) + 250*2^(1/2)*5^(1/2)*(- 5^(1/2)
 - 3)^(1/2))))*(- 5^(1/2) - 3)^(1/4)*1i)/4
```

sympy [A] time = 1.45, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(65536t^{8}+768t^{4}+1,\left(t\mapsto t\log\left(1024t^{5}+8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] -RootSum(65536*_t**8 + 768*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5 + 8*_t +
 x)))
```

3.22 $\int \frac{1-x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{x}{2\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {28, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{2\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]
/(4*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + Log[1 + Sqrt[2]*x + x
^2]/(8*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^4}{1+2x^4+x^8}\,dx &= \int \frac{1-x^4}{\left(1+x^4\right)^2}\,dx\\ &= \frac{x}{2\left(1+x^4\right)}+\frac{1}{2}\int \frac{1}{1+x^4}\,dx\\ &= \frac{x}{2\left(1+x^4\right)}+\frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx\\ &= \frac{x}{2\left(1+x^4\right)}+\frac{1}{8}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{8\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-}}{8\sqrt{2}}\\ &= \frac{x}{2\left(1+x^4\right)}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{4\sqrt{2}}-\\ &= \frac{x}{2\left(1+x^4\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{4\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+\right.}{8\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{1}{16}\left(\frac{8x}{x^4+1}-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
x^2])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 + 2*x^4 + x^8), x]
```

fricas [A] time = 1.23, size = 126, normalized size = 1.30

$$-\frac{4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{2}\left(x^4+1\right)\log\left(x^2+\sqrt{2}x+1\right)+\sqrt{2}\left(x^4+1\right)\log\left(x^2-\sqrt{2}x+1\right)-8x}{16\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x
 + 1) - 1) + 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqr
t(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^
4 + 1)*log(x^2 - sqrt(2)*x + 1) - 8*x)/(x^4 + 1)
```

giac [A] time = 0.30, size = 82, normalized size = 0.85

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$\frac{x}{2x^4+2}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{8}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+2*x^4+1),x)
```

```
[Out] 1/2/(x^4+1)*x+1/16*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))+1/8*2^(1
/2)*arctan(2^(1/2)*x-1)+1/8*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 1.56, size = 82, normalized size = 0.85

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

mupad [B] time = 1.62, size = 44, normalized size = 0.45

$$\frac{x}{2\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/8 - 1i/8) + x/(2*(x^4 + 1))
```

sympy [A] time = 0.18, size = 82, normalized size = 0.85

$$\frac{x}{2x^4+2} - \frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{16} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{16} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] x/(2*x**4 + 2) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)*log(x**2 +
sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + sqrt(2)*atan(sqrt(2)*x
+ 1)/8
```

3.23 $\int \frac{1-x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-}{\sqrt{3}}\right.$$

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]])/4 + ArcTan[Sqrt[3] - 2*x]/4 + (Sqrt[3]
*ArcTan[(1 + 2*x)/Sqrt[3]])/4 - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/
8 - Log[1 + x + x^2]/8 - (Sqrt[3]*Log[1 - Sqrt[3]*x + x^2])/8 + (Sqrt[3]*Lo
g[1 + Sqrt[3]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
```

```
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1+2x^2}{-1-x^2-x^4}\,dx\right) - \frac{1}{2}\int \frac{1-2x^2}{-1+x^2-x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1+x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1-x}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}-3x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+3x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}$$

$$= \frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx +$$

$$= \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{1}{8}\sqrt{3}\log\left(1-\sqrt{3}x+x^2\right) + \frac{1}{8}\sqrt{3}\log\left(1+\sqrt{3}x+x\right.$$

$$= -\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) +$$

Mathematica [C] time = 0.17, size = 129, normalized size = 0.92

$$\frac{1}{8}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)-2\sqrt{-2-2i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2\sqrt{-2+2i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] (-2*Sqrt[-2 - (2*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - 2*Sqrt[-2 + (2
*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2] - Log[1 + x
 + x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 + x^4 + x^8), x]
```

fricas [A] time = 1.16, size = 137, normalized size = 0.98

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{2}\arctan\left(-2x+\sqrt{3}+2\sqrt{x^2-\sqrt{3}x+1}\right)+\frac{1}{2}\arctan\left(-2x-\sqrt{3}+2\sqrt{x^2+\sqrt{3}x+1}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/8*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/8*sqrt(3)*log(x^2 - s
qrt(3)*x + 1) + 1/2*arctan(-2*x + sqrt(3) + 2*sqrt(x^2 - sqrt(3)*x + 1)) +
1/2*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x + 1)) - 1/8*log(x^2 + x
+ 1) + 1/8*log(x^2 - x + 1)
```

giac [A] time = 0.37, size = 108, normalized size = 0.77

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/8*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/8*sqrt(3)*log(x^2 - s
qrt(3)*x + 1) - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(3)) - 1/8
*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 109, normalized size = 0.78

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{4}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{8}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{8}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)+1/4*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/8*3^(1/2)*ln(x^2
-3^(1/2)*x+1)-1/4*arctan(2*x-3^(1/2))+1/8*3^(1/2)*ln(x^2+3^(1/2)*x+1)-1/4*a
rctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)+1/4*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{2}\int\frac{2x^2-1}{x^4-x^2+1}\,dx-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((2*x^2 - 1)/(x^4 - x^2 + 1), x) - 1/8*log(x^2 +
x + 1) + 1/8*log(x^2 - x + 1)
```

mupad [B] time = 0.19, size = 109, normalized size = 0.78

$$-\mathrm{atan}\left(\frac{54\sqrt{3}\,x}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{54\sqrt{3}\,x}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{-81+\sqrt{3}\,27\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\mathrm{atan}\left(\frac{\sqrt{3}\,x\,54\mathrm{i}}{81+\sqrt{3}\,27\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^4 + x^8 + 1),x)
```

```
[Out] atan((54*3^(1/2)*x)/(3^(1/2)*27i + 81))*(3^(1/2)/4 - 1i/4) - atan((54*3^(1/
2)*x)/(3^(1/2)*27i - 81))*(3^(1/2)/4 + 1i/4) + atan((3^(1/2)*x*54i)/(3^(1/2
)*27i - 81))*((3^(1/2)*1i)/4 - 1/4) - atan((3^(1/2)*x*54i)/(3^(1/2)*27i + 8
1))*((3^(1/2)*1i)/4 + 1/4)
```

sympy [C] time = 0.62, size = 148, normalized size = 1.06

$$-\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right)-\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right)-\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right)-\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right)-\mathrm{RootSum}\left(256t^4-16t^2+1,\left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+x**4+1),x)
```

```
[Out] -(-1/8 - sqrt(3)*I/8)*log(x + 1024*(-1/8 - sqrt(3)*I/8)**5) - (-1/8 + sqrt(
3)*I/8)*log(x + 1024*(-1/8 + sqrt(3)*I/8)**5) - (1/8 - sqrt(3)*I/8)*log(x +
 1024*(1/8 - sqrt(3)*I/8)**5) - (1/8 + sqrt(3)*I/8)*log(x + 1024*(1/8 + sqr
t(3)*I/8)**5) - RootSum(256*_t**4 - 16*_t**2 + 1, Lambda(_t, _t*log(1024*_t
**5 + x)))
```

3.24 $\int \frac{1-x^4}{1+x^8}\,dx$

Optimal. Leaf size=347

$$\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2\right.$$

Rubi [A] time = 0.27, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1414, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]])
 + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]]
) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]
]) + (Sqrt[(2 - Sqrt[2])/2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(
2 - Sqrt[2])/2]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])
/2]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log[1 +
Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1414

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-2*d*e, 2]}, Dist[d/(2*a), Int[(d - q*x^(n/2))/(d - q*x^(n/2) - e*x^
n), x], x] + Dist[d/(2*a), Int[(d + q*x^(n/2))/(d + q*x^(n/2) - e*x^n), x],
 x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 - a*e^2, 0] &&
IGtQ[n/2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1-x^4}{1+x^8}\,dx = \frac{1}{2}\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx + \frac{1}{2}\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}}$$

$$= -\left(\frac{1}{8}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \frac{(1-}{}$$

$$= \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{8}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{8}\sqrt{1+\frac{1}{\sqrt{2}}}$$

$$= -\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}$$

Mathematica [A] time = 0.16, size = 257, normalized size = 0.74

$$\frac{1}{8}\left(-\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)+\left(\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+2\left(\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x+\cos\left(\frac{\pi}{8}\right)\right)\right)+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)+2\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right)-x\csc\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 -
2*x*Sin[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]
]*(-Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(-Cos[Pi/8] + Sin
[Pi/8]) + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] + Sin[Pi/8]) + 2*A
rcTan[x*Sec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x*
Cos[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]]*(Cos[Pi/8
] + Sin[Pi/8]))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 + x^8), x]
```

fricas [B] time = 1.56, size = 991, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x
*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*(sq
rt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*(sqrt(sqrt(
2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) +
2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*(sqrt(sqrt(2) + 2) -
 sqrt(-sqrt(2) + 2))*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) -
 sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arc
tan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) +
 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(sqrt(2) +
 2)*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2)
+ 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) + sqrt(-sq
rt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-s
qrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(s
qrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + s
qrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*
sqrt(-sqrt(2) + 2)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x
*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) +
 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/32*
sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*s
qrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) +
 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2)
+ 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*lo
g(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2)
+ 1) + 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt(sqrt(
2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 - x*sq
rt(sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^
2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2
))*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

giac [A] time = 0.72, size = 247, normalized size = 0.71

$$\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
+ 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqr
t(2) + 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt
(-sqrt(2) + 2)) + 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) +
1) - 1/16*sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqr
t(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-2*sqrt(2
) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.01, size = 29, normalized size = 0.08

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8+1),x)
```

```
[Out] 1/8*sum((-_R^4+1)/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + 1), x)
```

mupad [B] time = 1.96, size = 312, normalized size = 0.90

$$-\ln\left(\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)^3\left(65536\,x-16384\sqrt{-2\sqrt{2}-4}+16384\sqrt{4-2\sqrt{2}}\right)-256\right)\left(\frac{\sqrt{-2\sqrt{2}-4}}{16}-\frac{\sqrt{4-2\sqrt{2}}}{16}\right)-\mathrm{atan}\left(-\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{2}\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(\frac{1}{2}+1\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{1}{4}-\frac{3}{4}\mathrm{i}\right)\right)\left(-2+\sqrt{2}\,(1-\mathrm{i})\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\frac{\mathrm{atan}\left(x\left(\sqrt{2}+2\right)^{3/2}\left(1-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,x\left(\sqrt{2}+2\right)^{3/2}\left(-\frac{3}{4}+\frac{1}{4}\mathrm{i}\right)\right)\left(\sqrt{2}\,(1+1\mathrm{i})-2\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}+\sqrt{2}\,\ln\left(x+\left(\sqrt{2}+2\right)^{3/2}\left(-1+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\left(\sqrt{2}+2\right)^{3/2}\left(\frac{3}{4}-\frac{1}{4}\mathrm{i}\right)\right)\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+2}}{16}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 + 1),x)
```

```
[Out] (atan(x*(2^(1/2) + 2)^(3/2)*(1/2 + 1i) - 2^(1/2)*x*(2^(1/2) + 2)^(3/2)*(1/4
 + 3i/4))*(2^(1/2)*(1 - 1i) - 2)*(2^(1/2) + 2)^(1/2)*1i)/8 - atan((x*1i)/(2
^(1/2) + 2)^(1/2) - (x*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*x*1i)/(2*(2^(1/2)
 - 2)^(1/2)) + (2^(1/2)*x*1i)/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2)*(2^(1/2) -
 2)^(1/2)*1i)/8 + (2^(1/2)*(2^(1/2) + 2)^(1/2)*1i)/8) - log(((- 2*2^(1/2) -
 4)^(1/2)/16 - (4 - 2*2^(1/2))^(1/2)/16)^3*(65536*x - 16384*(- 2*2^(1/2) -
4)^(1/2) + 16384*(4 - 2*2^(1/2))^(1/2)) - 256)*((- 2*2^(1/2) - 4)^(1/2)/16
- (4 - 2*2^(1/2))^(1/2)/16) + (atan(x*(2^(1/2) + 2)^(3/2)*(1 - 1i/2) - 2^(1
/2)*x*(2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*(2^(1/2)*(1 + 1i) - 2i)*(2^(1/2) +
2)^(1/2)*1i)/8 + 2^(1/2)*log(x - (2^(1/2) + 2)^(3/2)*(1 - 1i/2) + 2^(1/2)*(
2^(1/2) + 2)^(3/2)*(3/4 - 1i/4))*((2^(1/2) - 2)^(1/2)/16 + (2^(1/2) + 2)^(1
/2)/16)*1i
```

sympy [A] time = 2.75, size = 20, normalized size = 0.06

$$-\operatorname{RootSum}{\left(1048576 t^{8} + 1, \left(t \mapsto t \log{\left(4096 t^{5} - 4 t + x \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8+1),x)
```

```
[Out] -RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 - 4*_t + x)))
```

3.25 $\int \frac{1-x^4}{1-x^4+x^8} \, dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2\right.$$

Rubi [A] time = 0.28, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3]
)]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1-x^4+x^8}\,dx &= -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+}}{1+\sqrt{}}}{4\sqrt{3}} \\
&= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+} \\
&= \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8} \\
&= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - x^4 + x^8), x]
```

fricas [B] time = 1.63, size = 715, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

giac [A] time = 0.46, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.01, size = 44, normalized size = 0.12

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.67, size = 208, normalized size = 0.59

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.10, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}{\left(5308416 t^{8} - 2304 t^{4} + 1, \left(t \mapsto t \log{\left(9216 t^{5} - 8 t + x \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

3.26 $\int \frac{1-x^4}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=13

$$\frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 21, 212, 206, 203}

$$\frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ArcTan[x]/2 + ArcTanh[x]/2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx = \int \frac{1-x^4}{\left(-1+x^4\right)^2}\,dx$$
$$= -\int \frac{1}{-1+x^4}\,dx$$
$$= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$-\frac{1}{4}\log(1-x) + \frac{1}{4}\log(x+1) + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 2*x^4 + x^8),x]
[Out] ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - 2*x^4 + x^8),x]
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - 2*x^4 + x^8), x]
```

fricas [A] time = 1.42, size = 17, normalized size = 1.31

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x+1\right) - \frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

giac [B] time = 0.45, size = 19, normalized size = 1.46

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(|x+1|\right) - \frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\operatorname{arctanh}(x)}{2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-2*x^4+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)
```

maxima [A] time = 1.60, size = 17, normalized size = 1.31

$$\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.69

$$\frac{\operatorname{atan}(x)}{2}+\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atan(x)/2 + atanh(x)/2
```

sympy [B] time = 0.13, size = 17, normalized size = 1.31

$$-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + atan(x)/2
```

3.27 $\int \frac{1-x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=129

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqr
t[5])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1}{-1-x^2+x^4}\,dx\right) - \frac{1}{2}\int \frac{1}{-1+x^2+x^4}\,dx$$

$$= -\frac{\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}} - \frac{\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}} + \frac{\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}} + \frac{\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{2\sqrt{5}}$$

$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan[Sqrt[2/(1
 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/
Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqr
t[5])]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-3x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.44, size = 255, normalized size = 1.98

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(
2*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1) - 1/10*sqrt(10)*sqrt(5)*x*sqrt(sqrt(
5) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqr
t(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) - 1) - 1/10*sqrt(10)*sqrt(5)*x*
sqrt(sqrt(5) - 1)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5)
+ 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(
10)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) +
1)*log(sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqr
t(sqrt(5) + 1)*log(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x)
```

giac [A] time = 0.75, size = 147, normalized size = 1.14

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) + 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

maple [A]   time = 0.03, size = 110, normalized size = 0.85

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-3*x^4+1),x)
```

```
[Out] 1/5*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2
)/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2)/(-2+2*5
^(1/2))^(1/2)*arctanh(2/(-2+2*5^(1/2))^(1/2)*x)+1/5*5^(1/2)/(2+2*5^(1/2))^(
1/2)*arctan(2/(2+2*5^(1/2))^(1/2)*x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8-3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 3*x^4 + 1), x)
```

mupad [B]   time = 1.71, size = 269, normalized size = 2.09

$$-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2
))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) - 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

sympy [A]   time = 1.17, size = 51, normalized size = 0.40

$$-\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right)-\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(25600t^5-16t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] -RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 - 16*_t +
 x))) - RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5 -
16*_t + x)))
```

3.28 $\int \frac{1-x^4}{1-4x^4+x^8}\,dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

Rubi [A] time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sqrt[3])]) +
 ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])]) +
ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sqrt[3])])
+ ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}\,x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{2}\,x^2+x^4}\,dx$$

$$= -\frac{\int \frac{1}{-\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}}+\frac{\int \frac{1}{\sqrt{\frac{3}{2}}-\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}}-\frac{\int \frac{1}{-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}}+\frac{\int \frac{1}{\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+x^2}\,dx}{2\sqrt{6}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(-1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.33

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{\#1^7-2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] -1/8*RootSum[1 - 4*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-2*#1
^3 + #1^7) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-4x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - 4*x^4 + x^8), x]
```

fricas [B] time = 1.58, size = 302, normalized size = 1.83

$$-\frac{1}{6}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{6}\sqrt{6}\sqrt{x^2+\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}-\frac{1}{6}\sqrt{6}\left(\sqrt{3}x+3x\right)\left(-\sqrt{3}+2\right)^{\frac{3}{4}}\right)+\frac{1}{6}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{1}{6}\left(\sqrt{6}\sqrt{x^2-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-3\right)-\sqrt{6}\left(\sqrt{3}x-3x\right)\sqrt{\sqrt{3}+2}\right)\left(\sqrt{3}+2\right)^{\frac{1}{4}}\right)-\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right)+\frac{1}{24}\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-3\right)+6x\right)+\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)-\frac{1}{24}\sqrt{6}\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\log\left(-\sqrt{6}\left(\sqrt{3}+3\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}+6x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*(-sqrt(3) + 2)^(1/4)*arctan(1/6*sqrt(6)*sqrt(x^2 + (sqrt(3) +
2)*sqrt(-sqrt(3) + 2))*(sqrt(3) + 3)*(-sqrt(3) + 2)^(3/4) - 1/6*sqrt(6)*(sq
rt(3)*x + 3*x)*(-sqrt(3) + 2)^(3/4)) + 1/6*sqrt(6)*(sqrt(3) + 2)^(1/4)*arct
an(1/6*(sqrt(6)*sqrt(x^2 - sqrt(sqrt(3) + 2)*(sqrt(3) - 2))*sqrt(sqrt(3) +
2)*(sqrt(3) - 3) - sqrt(6)*(sqrt(3)*x - 3*x)*sqrt(sqrt(3) + 2))*(sqrt(3) +
2)^(1/4)) - 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(sqrt(6)*(sqrt(3) + 2)^(1/4
)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(sqrt(3) + 2)^(1/4)*log(-sqrt(6)*(sqr
t(3) + 2)^(1/4)*(sqrt(3) - 3) + 6*x) + 1/24*sqrt(6)*(-sqrt(3) + 2)^(1/4)*lo
g(sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) + 6*x) - 1/24*sqrt(6)*(-sqrt(3
) + 2)^(1/4)*log(-sqrt(6)*(sqrt(3) + 3)*(-sqrt(3) + 2)^(1/4) + 6*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 42, normalized size = 0.25

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)^4+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)^7-16\,\mathrm{RootOf}\left(\_Z^8-4\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-4*x^4+1),x)
```

```
[Out] 1/8*sum((-_R^4+1)/(_R^7-2*_R^3)*ln(-_R+x),_R=RootOf(_Z^8-4*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8-4\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-4*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

mupad [B] time = 0.18, size = 399, normalized size = 2.42

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\,\sqrt{6}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}}{80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{112\,\sqrt{3}\,\sqrt{6}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}}{3\,\left(80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}\right)}\right)\,{\left(\sqrt{3}+2\right)}^{1/4}}{12}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}\,64{}\mathrm{i}}{48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}}-\frac{\sqrt{3}\,\sqrt{6}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}\,112{}\mathrm{i}}{3\,\left(48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}\right)}\right)\,{\left(2-\sqrt{3}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{64\,\sqrt{6}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}}{48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}}-\frac{112\,\sqrt{3}\,\sqrt{6}\,x\,{\left(2-\sqrt{3}\right)}^{1/4}}{3\,\left(48\,\sqrt{3}\,\sqrt{2-\sqrt{3}}-80\,\sqrt{2-\sqrt{3}}\right)}\right)\,{\left(2-\sqrt{3}\right)}^{1/4}}{12}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}\,64{}\mathrm{i}}{80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\,\sqrt{6}\,x\,{\left(\sqrt{3}+2\right)}^{1/4}\,112{}\mathrm{i}}{3\,\left(80\,\sqrt{\sqrt{3}+2}+48\,\sqrt{3}\,\sqrt{\sqrt{3}+2}\right)}\right)\,{\left(\sqrt{3}+2\right)}^{1/4}\,1{}\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x)
```

```
[Out] (6^(1/2)*atan((6^(1/2)*x*(2 - 3^(1/2))^(1/4)*64i)/(48*3^(1/2)*(2 - 3^(1/2))
^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (3^(1/2)*6^(1/2)*x*(2 - 3^(1/2))^(1/4)*1
12i)/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2))))*(2 - 3^
(1/2))^(1/4)*1i)/12 - (6^(1/2)*atan((64*6^(1/2)*x*(2 - 3^(1/2))^(1/4))/(48*
3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/2))^(1/2)) - (112*3^(1/2)*6^(1/2
)*x*(2 - 3^(1/2))^(1/4))/(3*(48*3^(1/2)*(2 - 3^(1/2))^(1/2) - 80*(2 - 3^(1/
2))^(1/2))))*(2 - 3^(1/2))^(1/4))/12 + (6^(1/2)*atan((64*6^(1/2)*x*(3^(1/2)
 + 2)^(1/4))/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2)) + (1
12*3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4))/(3*(80*(3^(1/2) + 2)^(1/2) + 48*3
^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4))/12 - (6^(1/2)*atan((6^(1
/2)*x*(3^(1/2) + 2)^(1/4)*64i)/(80*(3^(1/2) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2
) + 2)^(1/2)) + (3^(1/2)*6^(1/2)*x*(3^(1/2) + 2)^(1/4)*112i)/(3*(80*(3^(1/2
) + 2)^(1/2) + 48*3^(1/2)*(3^(1/2) + 2)^(1/2))))*(3^(1/2) + 2)^(1/4)*1i)/12
```

sympy [A] time = 0.20, size = 26, normalized size = 0.16

$$-\mathrm{RootSum}\left(84934656t^8-36864t^4+1,\left(t\mapsto t\log\left(36864t^5-20t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] -RootSum(84934656*_t**8 - 36864*_t**4 + 1, Lambda(_t, _t*log(36864*_t**5 -
20*_t + x)))
```

3.29 $\int \frac{1-x^4}{1-5x^4+x^8}\,dx$

Optimal. Leaf size=169

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

Rubi [A] time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}\,x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7])] + ArcT
an[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sqrt[7])] + ArcTanh[Sq
rt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7])] + ArcTanh[Sqrt[
2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sqrt[7])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{3}x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{3}x^2+x^4}\,dx$$

$$= -\frac{\int \frac{1}{-\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}} - \frac{\int \frac{1}{\frac{\sqrt{3}}{2}-\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}} + \frac{\int \frac{1}{-\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}} + \frac{\int \frac{1}{\frac{\sqrt{3}}{2}+\frac{\sqrt{7}}{2}+x^2}\,dx}{2\sqrt{7}}$$

$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{-\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(-\sqrt{3}+\sqrt{7}\right)}} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}\,x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.34

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-5\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-5\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - 5*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-5*#1
^3 + 2*#1^7) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-5x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - 5*x^4 + x^8), x]
```

fricas [B] time = 1.81, size = 546, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/14*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5))*arctan(1/112*sqrt(1
4)*sqrt(4*x^2 + (sqrt(7)*sqrt(3)*sqrt(2) + 5*sqrt(2))*sqrt(-sqrt(7)*sqrt(3)
 + 5))*(sqrt(7)*sqrt(3)*sqrt(2) + 7*sqrt(2))*sqrt(-sqrt(7)*sqrt(3) + 5)*sqr
t(sqrt(2)*sqrt(-sqrt(7)*sqrt(3) + 5)) - 1/56*sqrt(14)*(sqrt(7)*sqrt(3)*sqrt
(2)*x + 7*sqrt(2)*x)*sqrt(-sqrt(7)*sqrt(3) + 5)*sqrt(sqrt(2)*sqrt(-sqrt(7)*
sqrt(3) + 5))) + 1/14*sqrt(14)*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*arct
an(1/112*(sqrt(14)*sqrt(4*x^2 - (sqrt(7)*sqrt(3)*sqrt(2) - 5*sqrt(2))*sqrt(
sqrt(7)*sqrt(3) + 5))*(sqrt(7)*sqrt(3)*sqrt(2) - 7*sqrt(2))*sqrt(sqrt(7)*sq
rt(3) + 5) - 2*sqrt(14)*(sqrt(7)*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(sqrt
(7)*sqrt(3) + 5))*sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))) - 1/56*sqrt(14)*
sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) - 7)*
sqrt(sqrt(2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(sqrt(2
)*sqrt(sqrt(7)*sqrt(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) - 7)*sqrt(sqrt(
2)*sqrt(sqrt(7)*sqrt(3) + 5)) + 28*x) + 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sq
rt(7)*sqrt(3) + 5))*log(sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-s
qrt(7)*sqrt(3) + 5)) + 28*x) - 1/56*sqrt(14)*sqrt(sqrt(2)*sqrt(-sqrt(7)*sqr
```

```
t(3) + 5))*log(-sqrt(14)*(sqrt(7)*sqrt(3) + 7)*sqrt(sqrt(2)*sqrt(-sqrt(7)*s
qrt(3) + 5)) + 28*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 44, normalized size = 0.26

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)^{4}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)^{7}-20\,\mathrm{RootOf}\left(\_Z^{8}-5\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-5*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-5*_R^3)*ln(-_R+x),_R=RootOf(_Z^8-5*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{4}-1}{x^{8}-5\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-5*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

mupad [B] time = 1.79, size = 483, normalized size = 2.86

$$\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right){\left(5-\sqrt{21}\right)}^{1/4}}{28}-\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}\,405{}\mathrm{i}}{2\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}-\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\,{\left(5-\sqrt{21}\right)}^{1/4}\,621{}\mathrm{i}}{14\left(243\sqrt{2}\sqrt{5-\sqrt{21}}-54\sqrt{2}\sqrt{21}\sqrt{5-\sqrt{21}}\right)}\right){\left(5-\sqrt{21}\right)}^{1/4}\,1{}\mathrm{i}}{28}+\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{405\,2^{3/4}\sqrt{7}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{621\,2^{3/4}\sqrt{7}\sqrt{21}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right){\left(\sqrt{21}+5\right)}^{1/4}}{28}-\frac{2^{3/4}\sqrt{7}\,\mathrm{atan}\left(\frac{2^{3/4}\sqrt{7}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}\,405{}\mathrm{i}}{2\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}+\frac{2^{3/4}\sqrt{7}\sqrt{21}\,x\,{\left(\sqrt{21}+5\right)}^{1/4}\,621{}\mathrm{i}}{14\left(243\sqrt{2}\sqrt{\sqrt{21}+5}+54\sqrt{2}\sqrt{21}\sqrt{\sqrt{21}+5}\right)}\right){\left(\sqrt{21}+5\right)}^{1/4}\,1{}\mathrm{i}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x)
```

```
[Out] (2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/4))/(2*(243*
2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))) -
 (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4))/(14*(243*2^(1/2)*(5
- 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1/2))))*(5 - 21^(1/
2))^(1/4))/28 - (2^(3/4)*7^(1/2)*atan((2^(3/4)*7^(1/2)*x*(5 - 21^(1/2))^(1/
4)*405i)/(2*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21
^(1/2))^(1/2))) - (2^(3/4)*7^(1/2)*21^(1/2)*x*(5 - 21^(1/2))^(1/4)*621i)/(1
4*(243*2^(1/2)*(5 - 21^(1/2))^(1/2) - 54*2^(1/2)*21^(1/2)*(5 - 21^(1/2))^(1
/2))))*(5 - 21^(1/2))^(1/4)*1i)/28 + (2^(3/4)*7^(1/2)*atan((405*2^(3/4)*7^(
1/2)*x*(21^(1/2) + 5)^(1/4))/(2*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(1
/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (621*2^(3/4)*7^(1/2)*21^(1/2)*x*(21^(
1/2) + 5)^(1/4))/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) + 54*2^(1/2)*21^(1/2
)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4))/28 - (2^(3/4)*7^(1/2)*atan(
(2^(3/4)*7^(1/2)*x*(21^(1/2) + 5)^(1/4)*405i)/(2*(243*2^(1/2)*(21^(1/2) + 5
)^(1/2) + 54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))) + (2^(3/4)*7^(1/2)*21^
(1/2)*x*(21^(1/2) + 5)^(1/4)*621i)/(14*(243*2^(1/2)*(21^(1/2) + 5)^(1/2) +
54*2^(1/2)*21^(1/2)*(21^(1/2) + 5)^(1/2))))*(21^(1/2) + 5)^(1/4)*1i)/28
```

sympy [**A**] time = 0.19, size = 26, normalized size = 0.15

$$-\operatorname{RootSum}\left(157351936t^8-62720t^4+1,\left(t\mapsto t\log\left(50176t^5-24t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] -RootSum(157351936*_t**8 - 62720*_t**4 + 1, Lambda(_t, _t*log(50176*_t**5 -
 24*_t + x)))
```

3.30 $\int \frac{1-x^4}{1-6x^4+x^8}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1419, 1093, 207, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTan[x/Sqrt[1 +
 Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqr
t[2*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])
])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1419

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0]
 || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^4}{1-6x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-2x^2+x^4}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+2x^2+x^4}\,dx\\ &= -\frac{\int \frac{1}{-1-\sqrt{2}+x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{1}{1-\sqrt{2}+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{-1+\sqrt{2}+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{1+\sqrt{2}+x^2}\,dx}{4\sqrt{2}}\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 114, normalized size = 0.91

$$\frac{\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[2]]*ArcTan
[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqrt[-1 + Sqrt[2]]] + S
qrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/(4*Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-6x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - 6*x^4 + x^8), x]
```

fricas [B] time = 1.40, size = 199, normalized size = 1.59

$$-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}+1}\arctan\left(-x\sqrt{\sqrt{2}+1}+\sqrt{x^2+\sqrt{2}-1}\sqrt{\sqrt{2}+1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}-1}\arctan\left(-x\sqrt{\sqrt{2}-1}+\sqrt{x^2+\sqrt{2}+1}\sqrt{\sqrt{2}-1}\right)+\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+x\right)+\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(sqrt(2) + 1)*arctan(-x*sqrt(sqrt(2) + 1) + sqrt(x^2 + sqr
t(2) - 1)*sqrt(sqrt(2) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(2) - 1)*arctan(-x*sqrt
(sqrt(2) - 1) + sqrt(x^2 + sqrt(2) + 1)*sqrt(sqrt(2) - 1)) + 1/16*sqrt(2)*s
qrt(sqrt(2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) - 1/16*sqrt(2)*sq
rt(sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + x) + 1/16*sqrt(2)*sq
rt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x) - 1/16*sqrt(2)*sqr
t(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + x)
```

giac [A] time = 0.63, size = 135, normalized size = 1.08

$$\frac{1}{8}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right)-\frac{1}{16}\sqrt{2\sqrt{2}-2}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)+\frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)-\frac{1}{16}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) + 1)) + 1/8*sqrt(2*sqrt(2) +
2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x + sqrt(
sqrt(2) + 1))) - 1/16*sqrt(2*sqrt(2) - 2)*log(abs(x - sqrt(sqrt(2) + 1))) +
 1/16*sqrt(2*sqrt(2) + 2)*log(abs(x + sqrt(sqrt(2) - 1))) - 1/16*sqrt(2*sqr
t(2) + 2)*log(abs(x - sqrt(sqrt(2) - 1)))
```

maple [A] time = 0.03, size = 90, normalized size = 0.72

$$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{8\sqrt{\sqrt{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-6*x^4+1),x)
```

```
[Out] 1/8*2^(1/2)/(2^(1/2)-1)^(1/2)*arctan(1/(2^(1/2)-1)^(1/2)*x)+1/8*2^(1/2)/(1+
2^(1/2))^(1/2)*arctanh(1/(1+2^(1/2))^(1/2)*x)+1/8*2^(1/2)/(1+2^(1/2))^(1/2)
*arctan(1/(1+2^(1/2))^(1/2)*x)+1/8*2^(1/2)/(2^(1/2)-1)^(1/2)*arctanh(1/(2^(
1/2)-1)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8-6\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-6*x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 6*x^4 + 1), x)
```

mupad [B] time = 0.20, size = 245, normalized size = 1.96

$$-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{x\,\sqrt{1-\sqrt{2}}\,4352\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{1-\sqrt{2}}\,3072\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\sqrt{1-\sqrt{2}}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{x\,\sqrt{-\sqrt{2}-1}\,4352\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{2}-1}\,3072\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\sqrt{-\sqrt{2}-1}\,1\mathrm{i}}{8}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{x\,\sqrt{\sqrt{2}-1}\,4352\mathrm{i}}{3072\,\sqrt{2}-4352}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}-1}\,3072\mathrm{i}}{3072\,\sqrt{2}-4352}\right)\sqrt{\sqrt{2}-1}\,1\mathrm{i}}{8}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{x\,\sqrt{\sqrt{2}+1}\,4352\mathrm{i}}{3072\,\sqrt{2}+4352}+\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+1}\,3072\mathrm{i}}{3072\,\sqrt{2}+4352}\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((x*(- 2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/2) + 4352) + (2^(1
/2)*x*(- 2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) + 4352))*(- 2^(1/2) - 1)^(
1/2)*1i)/8 - (2^(1/2)*atan((x*(1 - 2^(1/2))^(1/2)*4352i)/(3072*2^(1/2) - 43
52) - (2^(1/2)*x*(1 - 2^(1/2))^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(1 - 2^(
1/2))^(1/2)*1i)/8 + (2^(1/2)*atan((x*(2^(1/2) - 1)^(1/2)*4352i)/(3072*2^(1/
2) - 4352) - (2^(1/2)*x*(2^(1/2) - 1)^(1/2)*3072i)/(3072*2^(1/2) - 4352))*(
2^(1/2) - 1)^(1/2)*1i)/8 - (2^(1/2)*atan((x*(2^(1/2) + 1)^(1/2)*4352i)/(307
2*2^(1/2) + 4352) + (2^(1/2)*x*(2^(1/2) + 1)^(1/2)*3072i)/(3072*2^(1/2) + 4
352))*(2^(1/2) + 1)^(1/2)*1i)/8
```

sympy [A] time = 1.16, size = 51, normalized size = 0.41

$$-\operatorname{RootSum}\left(16384t^4-256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right)-\operatorname{RootSum}\left(16384t^4+256t^2-1,\left(t\mapsto t\log\left(65536t^5-28t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] -RootSum(16384*_t**4 - 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**5 - 28*_t
 + x))) - RootSum(16384*_t**4 + 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t**
5 - 28*_t + x)))
```

3.31 $\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=135

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1423, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) + ArcTan[(Sq
rt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log[1 - Sqrt[2 - Sqrt[3
]]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}\left(-1+\sqrt{3}\right)+\left(3-\sqrt{3}\right)x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}\left(-1+\sqrt{3}\right)+\left(-3+\sqrt{3}\right)x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}\,x-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}\,x-x^2}\,dx}{2\sqrt{2}} + \frac{1}{4}\left(-1+\sqrt{3}\right)\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx +$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{2\sqrt{2}} + \frac{1}{2}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{}{-2+}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2-\right.}{2\sqrt{}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.53

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{2\#1^4\log(x-\#1)+\sqrt{3}\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Sqrt[3]*Log[x - #1] + 2*Log[x -
 #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8), x]
```

fricas [A] time = 0.77, size = 104, normalized size = 0.77

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)x^3-\sqrt{2}x\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\left(\sqrt{3}\sqrt{2}+\sqrt{2}\right)x\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)x+2x^2+2}{\left(\sqrt{3}\sqrt{2}-\sqrt{2}\right)x-2x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(3)*sqrt(2) + sqrt(2))*x^3 - sqrt(2)*x) + 1/2*s
qrt(2)*arctan(1/2*(sqrt(3)*sqrt(2) + sqrt(2))*x) + 1/4*sqrt(2)*log(-((sqrt(
3)*sqrt(2) - sqrt(2))*x + 2*x^2 + 2)/((sqrt(3)*sqrt(2) - sqrt(2))*x - 2*x^2
 - 2))
```

giac [A] time = 0.49, size = 107, normalized size = 0.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\sqrt{2}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/2*sqr
t(2)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/4*sqrt(2)*lo
g(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/4*sqrt(2)*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1)
```

maple [C] time = 0.06, size = 47, normalized size = 0.35

$$\frac{\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4-1+\sqrt{3}\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-4\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(1/(2*_R^7-_R^3)*(-1+2*_R^4+3^(1/2))*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x^4+\sqrt{3}-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 2.24, size = 133, normalized size = 0.99

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{72\,\sqrt{2}\,x}{144\,\sqrt{3}-144\,\sqrt{3}\,x^2-288\,x^2+288}+\frac{72\,\sqrt{2}\,\sqrt{3}\,x}{144\,\sqrt{3}-144\,\sqrt{3}\,x^2-288\,x^2+288}\right)}{2}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{72\,\sqrt{2}\,x}{144\,\sqrt{3}+144\,\sqrt{3}\,x^2+288\,x^2+288}+\frac{72\,\sqrt{2}\,\sqrt{3}\,x}{144\,\sqrt{3}+144\,\sqrt{3}\,x^2+288\,x^2+288}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + 2*x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(1/2)*atan((72*2^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288
) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) - 144*3^(1/2)*x^2 - 288*x^2 + 288))
)/2 + (2^(1/2)*atanh((72*2^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*x^
2 + 288) + (72*2^(1/2)*3^(1/2)*x)/(144*3^(1/2) + 144*3^(1/2)*x^2 + 288*x^2
+ 288)))/2
```

sympy [A] time = 0.90, size = 163, normalized size = 1.21

$$\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(x\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)\right)+2\,\mathrm{atan}\left(x^3\left(\frac{\sqrt{6}}{1+\sqrt{3}}+\frac{2\sqrt{2}}{1+\sqrt{3}}\right)-\sqrt{2}\,x\right)\right)}{4}-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}\,x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}\,x\left(\frac{2}{\sqrt{3}+2}+\frac{2\sqrt{3}}{\sqrt{3}+2}\right)}{4}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x**4+3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(x*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3)))) + 2*at
an(x**3*(sqrt(6)/(1 + sqrt(3)) + 2*sqrt(2)/(1 + sqrt(3))) - sqrt(2)*x))/4 -
 sqrt(2)*log(x**2 - sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3) + 2))/4
 + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x*(2/(sqrt(3) + 2) + 2*sqrt(3)/(sqrt(3
) + 2))/4 + 1)/4
```

3.32 $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=164

$$-\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-}}\right.$$

Rubi [A] time = 0.09, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1423, 1161, 618, 204, 1164, 628}

$$-\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/2
+ (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/2
 - (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 + (Sqrt[2 + Sqr
t[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}-\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{4}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx - \frac{1}{4}\sqrt{2+\sqrt{3}}\int \frac{\sqrt{2-}}{-1-\sqrt{2}}$$

$$= -\frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) + \frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{2}\mathrm{S}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}} - \frac{1}{4}\sqrt{2+\sqrt{3}}\,\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) + \frac{1}{4}$$

Mathematica [C] time = 0.04, size = 72, normalized size = 0.44

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\sqrt{3}\#1^4\log(x-\#1)+\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4 + Sqrt[3]*Log[x
 - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.23, size = 111, normalized size = 0.68

$$\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(x^3\sqrt{\sqrt{3}+2}-x\sqrt{\sqrt{3}+2}\left(\sqrt{3}-1\right)\right)+\frac{1}{2}\sqrt{\sqrt{3}+2}\arctan\left(x\sqrt{\sqrt{3}+2}\right)+\frac{1}{4}\sqrt{\sqrt{3}+2}\log\left(-\frac{x\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-x^2-1}{x\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(sqrt(3) + 2)*arctan(x^3*sqrt(sqrt(3) + 2) - x*sqrt(sqrt(3) + 2)*(s
qrt(3) - 1)) + 1/2*sqrt(sqrt(3) + 2)*arctan(x*sqrt(sqrt(3) + 2)) + 1/4*sqrt
(sqrt(3) + 2)*log(-(x*sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - x^2 - 1)/(x*sqrt(sq
rt(3) + 2)*(sqrt(3) - 2) + x^2 + 1))
```

giac [A] time = 0.43, size = 123, normalized size = 0.75

$$\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2)
)) + 1/4*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sq
rt(2))) + 1/8*(sqrt(6) + sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1)
- 1/8*(sqrt(6) + sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.04, size = 62, normalized size = 0.38

$$\frac{\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+2\sqrt{3}\;\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+\left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{16\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x)
```

```
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(2*_R^4+2*3^(1/2)*_R^4+(1+3^(1/2))*(3^(1/2)-1))*ln(
-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(\sqrt{3}+1\right)+1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 2.19, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3^(1/2) + 1) + 1)/(x^8 - x^4 + 1),x)
```

```
[Out] 0
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**4*(1+3**(1/2)))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError
```

3.33 $\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=180

$$\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\,\tan^{-1}\left(\sqrt{\right.$$

Rubi [A] time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1423, 1161, 618, 204, 1164, 628}

$$\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)+\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])
/2 - (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[
3]]])/2 + (Sqrt[3*(2 - Sqrt[3])]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/4 - (S
qrt[3*(2 - Sqrt[3])]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c
*q*r), Int[(d*r - (d - e*q)*x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1
/(2*c*q*r), Int[(d*r + (d - e*q)*x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}\left(3-2\sqrt{3}\right)+\left(-6+3\sqrt{3}\right)x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}\left(3-2\sqrt{3}\right)+\left(6-3\sqrt{3}\right)x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)} \int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}\,x-x^2}\,dx + \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)} \int \frac{\sqrt{2-}}{-1+\sqrt{2-}}$$

$$= \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-}\right.$$

$$= \frac{1}{2}\sqrt{6-3\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{2}\sqrt{6-3\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2}{\sqrt{2-\sqrt{3}}}\right.$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.49

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\sqrt{3}\#1^4\log(x-\#1)-3\#1^4\log(x-\#1)-2\sqrt{3}\log(x-\#1)+3\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (3*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] - 3*Log[
x - #1]*#1^4 + Sqrt[3]*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.55, size = 141, normalized size = 0.78

$$-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}x^3\left(2\sqrt{3}+3\right)\sqrt{-3\sqrt{3}+6}-\frac{1}{3}x\left(\sqrt{3}+3\right)\sqrt{-3\sqrt{3}+6}\right)-\frac{1}{2}\sqrt{-3\sqrt{3}+6}\arctan\left(\frac{1}{3}x\left(2\sqrt{3}+3\right)\sqrt{-3\sqrt{3}+6}\right)+\frac{1}{4}\sqrt{-3\sqrt{3}+6}\log\left(\frac{3x^2-\sqrt{3}x\sqrt{-3\sqrt{3}+6}+3}{3x^2+\sqrt{3}x\sqrt{-3\sqrt{3}+6}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-3*sqrt(3) + 6)*arctan(1/3*x^3*(2*sqrt(3) + 3)*sqrt(-3*sqrt(3) +
6) - 1/3*x*(sqrt(3) + 3)*sqrt(-3*sqrt(3) + 6)) - 1/2*sqrt(-3*sqrt(3) + 6)*a
```

```
rctan(1/3*x*(2*sqrt(3) + 3)*sqrt(-3*sqrt(3) + 6)) + 1/4*sqrt(-3*sqrt(3) + 6
)*log((3*x^2 - sqrt(3)*x*sqrt(-3*sqrt(3) + 6) + 3)/(3*x^2 + sqrt(3)*x*sqrt(
-3*sqrt(3) + 6) + 3))
```

giac [A] time = 0.45, size = 131, normalized size = 0.73

$$\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6)
- sqrt(2))) + 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2))
 + 1) - 1/8*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

maple [C] time = 0.01, size = 62, normalized size = 0.34

$$\frac{\left(-6\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+2\sqrt{3}\ \mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4+\left(-3+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{16\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-8\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x)
```

```
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(-6*_R^4+2*3^(1/2)*_R^4+(-3+3^(1/2))*(3^(1/2)-1))*l
n(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(\sqrt{3}-3\right)-2\sqrt{3}+3}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 2.23, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3^(1/2) - 3) - 2*3^(1/2) + 3)/(x^8 - x^4 + 1),x)
```

```
[Out] 0
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x**4*(-3+3**(1/2))-2*3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError
```

3.34 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}} \, dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{a}\, d \tan^{-1}\left(\frac{\sqrt{c}\, x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e \log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1394, 774, 635, 205, 260}

$$-\frac{\sqrt{a}\, d \tan^{-1}\left(\frac{\sqrt{c}\, x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e \log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x^
2])/(2*c)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1394

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx &= \int \frac{x(e+dx)}{a+cx^2}\,dx\\
&= \frac{dx}{c}+\frac{\int \frac{-ad+cex}{a+cx^2}\,dx}{c}\\
&= \frac{dx}{c}-\frac{(ad)\int \frac{1}{a+cx^2}\,dx}{c}+e\int \frac{x}{a+cx^2}\,dx\\
&= \frac{dx}{c}-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$
-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{dx}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Log[a + c*x^
2])/(2*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x)/(c + a/x^2),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x)/(c + a/x^2), x]
```

fricas [A] time = 0.75, size = 108, normalized size = 2.20

$$
\left[\frac{d\sqrt{-\frac{a}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{a}{c}}-a}{cx^2+a}\right)+2\,dx+e\log\left(cx^2+a\right)}{2\,c},\,-\frac{2\,d\sqrt{\frac{a}{c}}\arctan\left(\frac{cx\sqrt{\frac{a}{c}}}{a}\right)-2\,dx-e\log\left(cx^2+a\right)}{2\,c}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/c)*log((c*x^2 - 2*c*x*sqrt(-a/c) - a)/(c*x^2 + a)) + 2*d*x
+ e*log(c*x^2 + a))/c, -1/2*(2*d*sqrt(a/c)*arctan(c*x*sqrt(a/c)/a) - 2*d*x
- e*log(c*x^2 + a))/c]
```

giac [A] time = 0.27, size = 43, normalized size = 0.88

$$
-\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{dx}{c}+\frac{e\log\left(cx^2+a\right)}{2\,c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="giac")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

maple [A] time = 0.01, size = 43, normalized size = 0.88

$$-\frac{ad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\, c}+\frac{dx}{c}+\frac{e \ln\left(c\, x^{2}+a\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x)/(c+a/x^2),x)
```

```
[Out] d*x/c+1/2*e*ln(c*x^2+a)/c-1/c*a*d/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

maxima [A] time = 1.62, size = 42, normalized size = 0.86

$$-\frac{ad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\, c}+\frac{dx}{c}+\frac{e \log\left(cx^{2}+a\right)}{2\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2),x, algorithm="maxima")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*log(c*x^2 + a)/c
```

mupad [B] time = 1.59, size = 39, normalized size = 0.80

$$\frac{e\, \ln\left(c\, x^{2}+a\right)}{2\, c}+\frac{d\, x}{c}-\frac{\sqrt{a}\, d\, \mathrm{atan}\left(\frac{\sqrt{c}\, x}{\sqrt{a}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x)/(c + a/x^2),x)
```

```
[Out] (e*log(a + c*x^2))/(2*c) + (d*x)/c - (a^(1/2)*d*atan((c^(1/2)*x)/a^(1/2)))/
c^(3/2)
```

sympy [B] time = 0.28, size = 112, normalized size = 2.29

$$\left(\frac{e}{2c}-\frac{d\sqrt{-ac^{3}}}{2c^{3}}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}-\frac{d\sqrt{-ac^{3}}}{2c^{3}}\right)+e}{d}\right)+\left(\frac{e}{2c}+\frac{d\sqrt{-ac^{3}}}{2c^{3}}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}+\frac{d\sqrt{-ac^{3}}}{2c^{3}}\right)+e}{d}\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x**2),x)
```

```
[Out] (e/(2*c) - d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(e/(2*c) - d*sqrt(-a*c**
3)/(2*c**3)) + e)/d) + (e/(2*c) + d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(
e/(2*c) + d*sqrt(-a*c**3)/(2*c**3)) + e)/d) + d*x/c
```

3.35 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} dx$

Optimal. Leaf size=86

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{dx}{c}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1393, 773, 634, 618, 206, 628}

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (d*x)/c - ((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*d - c*e)*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1393

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
```

```
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx &= \int \frac{x(e+dx)}{a+bx+cx^2} \, dx \\
&= \frac{dx}{c}+\frac{\int \frac{-ad+(-bd+ce)x}{a+bx+cx^2} \, dx}{c} \\
&= \frac{dx}{c}-\frac{(bd-ce)\int \frac{b+2cx}{a+bx+cx^2} \, dx}{2c^2}+\frac{\left(b^2d-2acd-bce\right)\int \frac{1}{a+bx+cx^2} \, dx}{2c^2} \\
&= \frac{dx}{c}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(b^2d-2acd-bce\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx,x,b+2cx\right)}{c^2} \\
&= \frac{dx}{c}-\frac{\left(b^2d-2acd-bce\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 86, normalized size = 1.00

$$
\frac{\frac{2\left(-2acd+b^2d-bce\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(ce-bd)\log(a+x(b+cx))+2cdx}{2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (2*c*d*x + (2*(b^2*d - 2*a*c*d - b*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (-(b*d) + c*e)*Log[a + x*(b + c*x)])/(2*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x)/(c + a/x^2 + b/x), x]
```

fricas [A] time = 1.20, size = 291, normalized size = 3.38

$$
\left[\frac{2\left(b^2c-4ac^2\right)dx+\left(bce-\left(b^2-2ac\right)d\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)-\left(\left(b^3-4abc\right)d-\left(b^2c-4ac^2\right)e\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^2-4ac^3\right)}, \frac{2\left(b^2c-4ac^2\right)dx+2\left(bce-\left(b^2-2ac\right)d\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)-\left(\left(b^3-4abc\right)d-\left(b^2c-4ac^2\right)e\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^2-4ac^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*d*x + (b*c*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*
x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*d*x + 2*(b*c*e - (b^2
 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
```

```
 - 4*a*c)) - ((b^3 - 4*a*b*c)*d - (b^2*c - 4*a*c^2)*e)*log(c*x^2 + b*x + a)
)/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.32, size = 85, normalized size = 0.99

$$\frac{dx}{c} - \frac{(bd - ce)\log\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{\left(b^2d - 2\,acd - bce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] d*x/c - 1/2*(b*d - c*e)*log(c*x^2 + b*x + a)/c^2 + (b^2*d - 2*a*c*d - b*c*e
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 161, normalized size = 1.87

$$-\frac{2ad\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} + \frac{b^2d\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{be\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} - \frac{bd\ln\left(cx^2+bx+a\right)}{2c^2} + \frac{dx}{c} + \frac{e\ln\left(cx^2+bx+a\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x)/(c+a/x^2+b/x),x)
```

```
[Out] 1/c*d*x-1/2/c^2*ln(c*x^2+b*x+a)*b*d+1/2/c*ln(c*x^2+b*x+a)*e-2/c/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d+1/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.77, size = 127, normalized size = 1.48

$$\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(d\,b^3-e\,b^2\,c-4\,a\,d\,b\,c+4\,a\,e\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} + \frac{d\,x}{c} - \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-d\,b^2+c\,e\,b+2\,a\,c\,d\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x)/(c + a/x^2 + b/x),x)
```

```
[Out] (log(a + b*x + c*x^2)*(b^3*d + 4*a*c^2*e - b^2*c*e - 4*a*b*c*d))/(2*(4*a*c^
3 - b^2*c^2)) + (d*x)/c - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*a*c*d - b^2*d + b*c*e))/(c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.37, size = 423, normalized size = 4.92

$$\left(-\frac{\sqrt{-4ac+b^2}\,\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right)\log\left(x + \frac{-abd - 4ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right) + 2ace + b^2c\left(-\frac{\sqrt{-4ac+b^2}\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right)}{2acd - b^2d + bce}\right) + \left(\frac{\sqrt{-4ac+b^2}\,\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right)\log\left(x + \frac{-abd - 4ac^2\left(\frac{\sqrt{-4ac+b^2}\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right) + 2ace + b^2c\left(\frac{\sqrt{-4ac+b^2}\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right)}{2acd - b^2d + bce}\right) + \frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x)/(c+a/x**2+b/x),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) -
(b*d - c*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(-sqrt(-4*a*c + b**2)*(2*a
*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*
a*c*e + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*
a*c - b**2)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + (sqrt(-
4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c
*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*d - b*
*2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)) + 2*a*c*e + b
**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2
)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + d*x/c
```

3.36 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

Optimal. Leaf size=253

$$\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,c^{5/4}}+\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)}{2\sqrt{}}$$

Rubi [A] time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1394, 1280, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,c^{5/4}}+\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\,c^{5/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) + ((Sqrt[a]*d + Sqrt[c]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4
)*c^(5/4)) - ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1394

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx &= \int \frac{x^2\left(e+dx^2\right)}{a+cx^4}\,dx\\
&= \frac{dx}{c}-\frac{\int \frac{ad-cex^2}{a+cx^4}\,dx}{c}\\
&= \frac{dx}{c}-\frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}-\frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}\\
&= \frac{dx}{c}-\frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c}-\frac{\left(\frac{\sqrt{a}\,d}{\sqrt{c}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}\cdots}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\cdots}{\cdots}}}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}\cdots\\
&= \frac{dx}{c}+\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\cdots\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}\cdots\\
&= \frac{dx}{c}+\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}-\frac{\left(\sqrt{a}\,d-\sqrt{c}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{c}\,e\right)\log\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 293, normalized size = 1.16

$$
\frac{\left(a^{5/4}\sqrt{c}\,d+a^{3/4}ce\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,ac^{7/4}}-\frac{\left(a^{5/4}\sqrt{c}\,d+a^{3/4}ce\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,ac^{7/4}}+\frac{\left(a^{3/4}ce-a^{5/4}\sqrt{c}\,d\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}\,x-\sqrt{2}\,\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{a}}\right)}{2\sqrt{2}\,ac^{7/4}}+\frac{\left(a^{3/4}ce-a^{5/4}\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}+2\sqrt[4]{c}\,x}{\sqrt{2}\,\sqrt[4]{a}}\right)}{2\sqrt{2}\,ac^{7/4}}+\frac{dx}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((-(a^(5/4)*Sqrt[c]*d) + a^(3/4)*c*e)*ArcTan[(-(Sqrt[2]*a^(1/4))
+ 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*a*c^(7/4)) + ((-(a^(5/4)*Sqrt
[c]*d) + a^(3/4)*c*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/
```

```
4))])/(2*Sqrt[2]*a*c^(7/4)) + ((a^(5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a*c^(7/4)) - ((a^(
5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a*c^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x^2)/(c + a/x^4), x]
```

fricas [B] time = 1.36, size = 754, normalized size = 2.98

$$\frac{c\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+2de}{c^2}}\log\left(-(a^2d^4-c^2e^4)x+\left(ac^4e\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+a^2cd^3-ac^2de^2\right)\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+2de}{c^2}}\right)-c\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+2de}{c^2}}\log\left(-(a^2d^4-c^2e^4)x-\left(ac^4e\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+a^2cd^3-ac^2de^2\right)\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}+2de}{c^2}}\right)-c\sqrt{-\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}-2de}{c^2}}\log\left(-(a^2d^4-c^2e^4)x+\left(ac^4e\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}-a^2cd^3+ac^2de^2\right)\sqrt{-\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}-2de}{c^2}}\right)+c\sqrt{-\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}-2de}{c^2}}\log\left(-(a^2d^4-c^2e^4)x-\left(ac^4e\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}-a^2cd^3+ac^2de^2\right)\sqrt{-\frac{c^2\sqrt{-\frac{a^2d^4-2acd^2e^2+c^2e^4}{ac^5}}-2de}{c^2}}\right)+4dx}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e
)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2
 + c^2*e^4)/(a*c^5)) + a^2*c*d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 -
2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)) - c*sqrt((c^2*sqrt(-(a^2*d
^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^
4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + a^2*c*
d^3 - a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c
^5)) + 2*d*e)/c^2)) - c*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4
)/(a*c^5)) - 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqrt(-(a^2*d
^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^
2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + c*sqr
t(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)*lo
g(-(a^2*d^4 - c^2*e^4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^
4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^
2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + 4*d*x)/c
```

giac [A] time = 0.35, size = 247, normalized size = 0.98

$$\frac{dx}{c}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}acd-\left(ac^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{4ac^3}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}acd-\left(ac^3\right)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{4ac^3}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}acd+\left(ac^3\right)^{\frac34}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}acd+\left(ac^3\right)^{\frac34}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="giac")
```

```
[Out] d*x/c - 1/4*sqrt(2)*((a*c^3)^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/4*sqrt(2)*((a*c^3)
^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)*((a*
c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c^3)
```

maple [A] time = 0.01, size = 266, normalized size = 1.05

$$\frac{dx}{c}-\frac{\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac14}}-1\right)}{4c}-\frac{\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac14}}+1\right)}{4c}-\frac{\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8c}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac14}}-1\right)}{4\left(\frac{a}{c}\right)^{\frac14}c}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac14}}+1\right)}{4\left(\frac{a}{c}\right)^{\frac14}c}+\frac{\sqrt{2}\,e\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{a}{c}\right)^{\frac14}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^2)/(c+a/x^4),x)
```

```
[Out] 1/c*d*x-1/4/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/c*d
*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^
(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)+1/8/c*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/c*e/(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/c*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x-1)
```

maxima [**A**] time = 1.30, size = 240, normalized size = 0.95

$$\frac{dx}{c}-\frac{\frac{2\sqrt{2}\left(a\sqrt{c}d-\sqrt{a}ce\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(a\sqrt{c}d-\sqrt{a}ce\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(a\sqrt{c}d+\sqrt{a}ce\right)\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(a\sqrt{c}d+\sqrt{a}ce\right)\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4),x, algorithm="maxima")
```

```
[Out] d*x/c - 1/8*(2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(a*sqrt(c)*d - sqrt(a)*c*e)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*lo
g(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sq
rt(2)*(a*sqrt(c)*d + sqrt(a)*c*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)
*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

mupad [**B**] time = 0.31, size = 555, normalized size = 2.19

$$\frac{dx}{c}-2\,\mathrm{atanh}\left(\frac{8a^{2}cd^{2}x\sqrt{\frac{d^{2}\sqrt{-ac^{5}}}{16c^{5}}+\frac{de}{8c^{2}}-\frac{e^{2}\sqrt{-ac^{5}}}{16ac^{4}}}}{2a^{2}d^{2}e-2ace^{3}+\frac{2a^{2}d^{3}\sqrt{-ac^{5}}}{c^{3}}-\frac{2ade^{2}\sqrt{-ac^{5}}}{c^{2}}}-\frac{8ac^{2}e^{2}x\sqrt{\frac{d^{2}\sqrt{-ac^{5}}}{16c^{5}}+\frac{de}{8c^{2}}-\frac{e^{2}\sqrt{-ac^{5}}}{16ac^{4}}}}{2a^{2}d^{2}e-2ace^{3}+\frac{2a^{2}d^{3}\sqrt{-ac^{5}}}{c^{3}}-\frac{2ade^{2}\sqrt{-ac^{5}}}{c^{2}}}\right)\sqrt{\frac{ad^{2}\sqrt{-ac^{5}}-ce^{2}\sqrt{-ac^{5}}+2ac^{3}de}{16ac^{5}}}-2\,\mathrm{atanh}\left(\frac{8a^{2}cd^{2}x\sqrt{\frac{de}{8c^{2}}-\frac{d^{2}\sqrt{-ac^{5}}}{16c^{5}}+\frac{e^{2}\sqrt{-ac^{5}}}{16ac^{4}}}}{2a^{2}d^{2}e-2ace^{3}-\frac{2a^{2}d^{3}\sqrt{-ac^{5}}}{c^{3}}+\frac{2ade^{2}\sqrt{-ac^{5}}}{c^{2}}}-\frac{8ac^{2}e^{2}x\sqrt{\frac{de}{8c^{2}}-\frac{d^{2}\sqrt{-ac^{5}}}{16c^{5}}+\frac{e^{2}\sqrt{-ac^{5}}}{16ac^{4}}}}{2a^{2}d^{2}e-2ace^{3}-\frac{2a^{2}d^{3}\sqrt{-ac^{5}}}{c^{3}}+\frac{2ade^{2}\sqrt{-ac^{5}}}{c^{2}}}\right)\sqrt{\frac{ce^{2}\sqrt{-ac^{5}}-ad^{2}\sqrt{-ac^{5}}+2ac^{3}de}{16ac^{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^2)/(c + a/x^4),x)
```

```
[Out] (d*x)/c - 2*atanh((8*a^2*c*d^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*
c^2) - (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (
2*a^2*d^3*(-a*c^5)^(1/2))/c^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*
e^2*x*((d^2*(-a*c^5)^(1/2))/(16*c^5) + (d*e)/(8*c^2) - (e^2*(-a*c^5)^(1/2))
/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*e^3 + (2*a^2*d^3*(-a*c^5)^(1/2))/c
^3 - (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((a*d^2*(-a*c^5)^(1/2) - c*e^2*(-a*c^
5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2) - 2*atanh((8*a^2*c*d^2*x*((d*e)/(
8*c^2) - (d^2*(-a*c^5)^(1/2))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(
1/2))/(2*a^2*d^2*e - 2*a*c*e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^
2*(-a*c^5)^(1/2))/c^2) - (8*a*c^2*e^2*x*((d*e)/(8*c^2) - (d^2*(-a*c^5)^(1/2
))/(16*c^5) + (e^2*(-a*c^5)^(1/2))/(16*a*c^4))^(1/2))/(2*a^2*d^2*e - 2*a*c*
e^3 - (2*a^2*d^3*(-a*c^5)^(1/2))/c^3 + (2*a*d*e^2*(-a*c^5)^(1/2))/c^2))*((c
*e^2*(-a*c^5)^(1/2) - a*d^2*(-a*c^5)^(1/2) + 2*a*c^3*d*e)/(16*a*c^5))^(1/2)
```

sympy [**A**] time = 0.70, size = 109, normalized size = 0.43

$$\operatorname{RootSum}\left(256t^{4}ac^{5}-64t^{2}ac^{3}de+a^{2}d^{4}+2acd^{2}e^{2}+c^{2}e^{4},\left(t\mapsto t\log\left(x+\frac{-64t^{3}ac^{4}e-4ta^{2}cd^{3}+12tac^{2}de^{2}}{a^{2}d^{4}-c^{2}e^{4}}\right)\right)\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**2)/(c+a/x**4),x)
```

```
[Out] RootSum(256*_t**4*a*c**5 - 64*_t**2*a*c**3*d*e + a**2*d**4 + 2*a*c*d**2*e**
2 + c**2*e**4, Lambda(_t, _t*log(x + (-64*_t**3*a*c**4*e - 4*_t*a**2*c*d**3
 + 12*_t*a*c**2*d*e**2)/(a**2*d**4 - c**2*e**4)))) + d*x/c
```

3.37 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=208

$$-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

Rubi [A] time = 0.54, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1393, 1279, 1166, 205}

$$-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1393

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx &= \int \frac{x^2\left(e+dx^2\right)}{a+bx^2+cx^4}\,dx \\
&= \frac{dx}{c}-\frac{\int \frac{ad+(bd-ce)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}}{2c} \\
&= \frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4a}}}\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 251, normalized size = 1.21

$$
-\frac{\left(bd\sqrt{b^2-4ac}-ce\sqrt{b^2-4ac}+2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd\sqrt{b^2-4ac}-ce\sqrt{b^2-4ac}-2acd+b^2d-bce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + b*c*e - c*Sqrt[b^2
 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*d - 2*a*
c*d + b*Sqrt[b^2 - 4*a*c]*d - b*c*e - c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x^2)/(c + a/x^4 + b/x^2), x]
```

fricas [B] time = 1.67, size = 2540, normalized size = 12.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^
2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^
2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*
e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*
b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x + sqrt
(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b
^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*
```

```
e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^
3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))
*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^
3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 -
4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*
a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d
*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*
e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 -
 (b^3 + a*b*c)*d^3*e)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*
(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c
^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*
d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(
b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2
 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*
c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c
^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*
d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c
^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x + sqrt(1/2)*((b^4 - 5
*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^
3)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c
^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*
d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e
^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sq
rt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c -
 a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*
(b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^
2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2
*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*
d^3*e)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*
c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*
c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c
^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c
^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2
)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2
*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e
^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*d*x)/c
```

giac [B] time = 3.76, size = 3183, normalized size = 15.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] d*x/c + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*d -
(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
 - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 -
 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2
*c^4)*d*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^
5)*d + (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16
*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16
*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4
*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*d - (2*b^4*c^3 - 16*a*b^2*c^4 +
32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8
*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*
b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 +
2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*d*abs(c) - (2*b^5*c^4
- 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2
*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d + (2*b^4*c^5 - 8*a*b^2*
c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4
```

```
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*
b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))
/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b
^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B]   time = 0.03, size = 560, normalized size = 2.69

$$\frac{\sqrt{2}\,ad\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2d\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,b^2d\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}+\frac{\sqrt{2}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,be\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,bd\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\,c}-\frac{\sqrt{2}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^2)/(c+a/x^4+b/x^2),x)
```

```
[Out] 1/c*d*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^2*d+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/2/c*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b*d+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*d-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/2/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{dx}{c}+\frac{-\int \frac{(bd-ce)x^2+ad}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^2)/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate(-((b*d - c*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B]   time = 2.85, size = 6366, normalized size = 30.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^2)/(c + a/x^4 + b/x^2),x)
```

```
[Out] (d*x)/c - atan(((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^
2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*
a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4
*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 -
 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4*d^2 - 2*a*c^3*e^2
 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*
```

```
e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2
 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^
2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*
c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4*d^2
- 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 +
 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2
*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c
^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*
e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 - b^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*
a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e
^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*d
^3 + a*b^2*d^2*e + a^2*c*d^2*e - 2*a*b*c*d*e^2))/c + (((16*a^2*c^3*d - 4*a*
b^2*c^2*d)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^
2 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2
*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) + (2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3
*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 - b^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*d^2 -
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 - c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^
2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*
```

```
a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4
*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 -
 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4*d^2 - 2*a*c^3*e^2
 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*
e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2
 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*d - 4*a*b^2*c^2*d)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^
2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*
c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4*d^2
- 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*a*b^2*c*d^2 +
 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2
*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e -
7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c
^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*d - 4*a*b^2*c^2*d)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*
e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^2 + b^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 4*
a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e
^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*d
^3 + a*b^2*d^2*e + a^2*c*d^2*e - 2*a*b*c*d*e^2))/c + (((16*a^2*c^3*d - 4*a*
b^2*c^2*d)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5*d^
2 + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2
*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) + (2*x*(b^4*d^2 - 2*a*c^3*e^2 + 2*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3
*c*d*e - 4*a*b^2*c*d^2 + 6*a*b*c^2*d*e))/c)*(-(b^5*d^2 + b^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*d^2 +
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^2*e^2 + c^2*e^2*(-(4*a*c - b^2)^3
```

```
)^(1/2) + 12*a^2*b*c^2*d^2 - 2*b^4*c*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b*c^3*e^2 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**2)/(c+a/x**4+b/x**2),x)
```

```
[Out] Timed out
```

3.38 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}} \, dx$

Optimal. Leaf size=311

$$\frac{\left(\sqrt{3}\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12\sqrt[3]{a}\,c^{7/6}}-\frac{\left(\sqrt{3}\sqrt{a}\,d-\sqrt{c}\,e\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12\sqrt[3]{a}\,c^{7/6}}+\frac{\left(\sqrt{a}\right.}{}$$

Rubi [A] time = 0.29, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1394, 1503, 1416, 635, 203, 260, 634, 617, 204, 628}

$$\frac{\left(\sqrt{3}\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12\sqrt[3]{a}\,c^{7/6}}-\frac{\left(\sqrt{3}\sqrt{a}\,d-\sqrt{c}\,e\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12\sqrt[3]{a}\,c^{7/6}}+\frac{\left(\sqrt{a}\,d-\sqrt{3}\sqrt{c}\,e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}\,c^{7/6}}-\frac{\left(\sqrt{a}\,d+\sqrt{3}\sqrt{c}\,e\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)}{6\sqrt[3]{a}\,c^{7/6}}-\frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((Sqrt[a]*d
 - Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6*a^(1/3)*c
^(7/6)) - ((Sqrt[a]*d + Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a
^(1/6)])/(6*a^(1/3)*c^(7/6)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^
(2/3)) + ((Sqrt[3]*Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(
1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(7/6)) - ((Sqrt[3]*Sqrt[a]*d - Sqrt[c]
*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(
7/6))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1394

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1416

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rule 1503

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + c*x^(2*n))^(p +
1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx &= \int \frac{x^3\left(e+dx^3\right)}{a+cx^6}\,dx \\
&= \frac{dx}{c} - \frac{\int \frac{ad-cex^3}{a+cx^6}\,dx}{c} \\
&= \frac{dx}{c} - \frac{\int \frac{2a^{2/3}\sqrt[3]{c}\,d-\left(\sqrt{3}\sqrt{a}\sqrt{c}\,d+ce\right)x}{1-\frac{\sqrt{3}\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{2a^{2/3}\sqrt[3]{c}\,d+\left(\sqrt{3}\sqrt{a}\sqrt{c}\,d-ce\right)x}{1+\frac{\sqrt{3}\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{6a^{2/3}c^{4/3}} - \frac{\int \frac{a^{2/3}\sqrt[3]{c}\,d+cex}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}c^{4/3}} \\
&= \frac{dx}{c} - \frac{d\int \frac{1}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3c} - \frac{e\int \frac{x}{1+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{3a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt{3}\sqrt{a}\,d-\sqrt{c}\,e\right)\int \frac{\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt[6]{a}}+\frac{2\sqrt[3]{c}\,x}{\sqrt[3]{a}}}{1+\frac{\sqrt{3}\sqrt[6]{c}\,x}{\sqrt[6]{a}}+\frac{\sqrt[3]{c}\,x^2}{\sqrt[3]{a}}}\,dx}{12\sqrt[3]{a}\,c^{7/6}} + \left(\sqrt{3}\sqrt{a}\,d+\sqrt{c}\cdots\right. \\
&= \frac{dx}{c} - \frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}} + \frac{\left(\sqrt{3}\sqrt{a}\,d+\sqrt{c}\,e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\cdots\right.}{12\sqrt[3]{a}\,c^{7/6}} \\
&= \frac{dx}{c} - \frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(\sqrt{a}\,d-\sqrt{3}\sqrt{c}\,e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}\,c^{7/6}} - \frac{\left(\sqrt{a}\,d+\sqrt{3}\sqrt{c}\,e\right)\tan^{-1}\left(\sqrt{3}\cdots\right.}{6\sqrt[3]{a}\,c^{7/6}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 346, normalized size = 1.11

$$
-\frac{\left(-\sqrt{3}a^{7/6}\sqrt{c}\,d-a^{2/3}ce\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12ac^{5/3}} - \frac{\left(\sqrt{3}a^{7/6}\sqrt{c}\,d-a^{2/3}ce\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\,x+\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{12ac^{5/3}} + \frac{\left(\sqrt{3}a^{2/3}ce-a^{7/6}\sqrt{c}\,d\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}\,x-\sqrt{3}\sqrt[6]{a}}{\sqrt[6]{a}}\right)}{6ac^{5/3}} + \frac{\left(a^{7/6}\left(-\sqrt{c}\right)d-\sqrt{3}a^{2/3}ce\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{6ac^{5/3}} - \frac{\sqrt[6]{a}\,d\tan^{-1}\left(\frac{\sqrt[6]{c}\,x}{\sqrt[6]{a}}\right)}{3c^{7/6}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}\,x^2\right)}{6\sqrt[3]{a}\,c^{2/3}} + \frac{dx}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((-(a^(7/6)
*Sqrt[c]*d) + Sqrt[3]*a^(2/3)*c*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x
)/a^(1/6)])/(6*a*c^(5/3)) + ((-(a^(7/6)*Sqrt[c]*d) - Sqrt[3]*a^(2/3)*c*e)*A
rcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/3)) - (e*Log[a^(1
/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((-(Sqrt[3]*a^(7/6)*Sqrt[c]*d) -
a^(2/3)*c*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*
c^(5/3)) - ((Sqrt[3]*a^(7/6)*Sqrt[c]*d - a^(2/3)*c*e)*Log[a^(1/3) + Sqrt[3]
*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x^3)/(c + a/x^6), x]
```

fricas [B] time = 2.13, size = 3169, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/
(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*arctan(1/3*(2*(sqrt(3)*(a^2*c^
6*d^2 - a*c^7*e^2)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7))
 - 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3))*sqrt(((a^3*d^7 - a^2*c*d^5*
e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 + (2*a^2*c^6*d*e*sqrt(-(a^2*d^6 -
6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + a^3*c^2*d^5 - 4*a^2*c^3*d^3*e^2 +
 3*a*c^4*d*e^4)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*
c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(2/3) + ((a^2*c^5*d^2*e + a*c^6*e^3)*x*
sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + (a^3*c*d^6 - 2*a
^2*c^2*d^4*e^2 - 3*a*c^3*d^2*e^4)*x)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3))/(a^3*d^7 -
a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6))*((a*c^3*sqrt(-(a^2*d^6 - 6*
a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(2/3) -
 2*(sqrt(3)*(a^2*c^6*d^2 - a*c^7*e^2)*x*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*
c^2*d^2*e^4)/(a*c^7)) - 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3)*x)*((a*
c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e -
c*e^3)/(a*c^3))^(2/3) + sqrt(3)*(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4
- 3*c^3*d*e^6))/(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6))
- 4*sqrt(3)*c*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c
^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)*arctan(1/3*(2*(sqrt(3)*(a^2*c^6*d^
2 - a*c^7*e^2)*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 2
*sqrt(3)*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3))*sqrt(((a^3*d^7 - a^2*c*d^5*e^2
- 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 - (2*a^2*c^6*d*e*sqrt(-(a^2*d^6 - 6*a*
c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^3*c^2*d^5 + 4*a^2*c^3*d^3*e^2 - 3*a
*c^4*d*e^4)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7
)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(2/3) - ((a^2*c^5*d^2*e + a*c^6*e^3)*x*sqr
t(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - (a^3*c*d^6 - 2*a^2*
c^2*d^4*e^2 - 3*a*c^3*d^2*e^4)*x)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 +
 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3))/(a^3*d^7 - a^
2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6))*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a
*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(2/3) -
2*(sqrt(3)*(a^2*c^6*d^2 - a*c^7*e^2)*x*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c
^2*d^2*e^4)/(a*c^7)) + 2*sqrt(3)*(a^2*c^3*d^4*e - 3*a*c^4*d^2*e^3)*x)*(-(a*
c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e +
c*e^3)/(a*c^3))^(2/3) - sqrt(3)*(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4
- 3*c^3*d*e^6))/(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6))
+ c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*
d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e
^4 - 3*c^3*d*e^6)*x^2 - (2*a^2*c^6*d*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c
^2*d^2*e^4)/(a*c^7)) + a^3*c^2*d^5 - 4*a^2*c^3*d^3*e^2 + 3*a*c^4*d*e^4)*((a
*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e -
 c*e^3)/(a*c^3))^(2/3) - ((a^2*c^5*d^2*e + a*c^6*e^3)*x*sqrt(-(a^2*d^6 - 6*
a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + (a^3*c*d^6 - 2*a^2*c^2*d^4*e^2 - 3*
a*c^3*d^2*e^4)*x)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(
a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) + c*(-(a*c^3*sqrt(-(a^2*d^6 -
6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)
*log(-(a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 + (2*a^
2*c^6*d*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^3*c^
2*d^5 + 4*a^2*c^3*d^3*e^2 - 3*a*c^4*d*e^4)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*
d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(2/3) + ((a
^2*c^5*d^2*e + a*c^6*e^3)*x*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)
/(a*c^7)) - (a^3*c*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a*c^3*d^2*e^4)*x)*(-(a*c^3*s
qrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3
)/(a*c^3))^(1/3)) - 2*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*
e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3
*e^2 - 3*c^2*d*e^4)*x + (a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2
*e^4)/(a*c^7)) + a^2*c*d^4 - 3*a*c^2*d^2*e^2)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*
c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) -
2*c*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a
```

```
*d^2*e + c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)
*x - (a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^
2*c*d^4 + 3*a*c^2*d^2*e^2)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*
d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)) - 12*d*x)/c
```

giac [A] time = 0.53, size = 295, normalized size = 0.95

$$-\frac{|c|e\log\left(x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(ac^{5}\right)^{\frac{1}{3}}}+\frac{dx}{c}-\frac{\left(ac^{5}\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3c^{2}}-\frac{\left(\left(ac^{5}\right)^{\frac{1}{6}}ac^{2}d+\sqrt{3}\left(ac^{5}\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^{4}}-\frac{\left(\left(ac^{5}\right)^{\frac{1}{6}}ac^{2}d-\sqrt{3}\left(ac^{5}\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6ac^{4}}-\frac{\left(\sqrt{3}\left(ac^{5}\right)^{\frac{1}{6}}ac^{2}d-\left(ac^{5}\right)^{\frac{2}{3}}e\right)\log\left(x^{2}+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^{4}}+\frac{\left(\sqrt{3}\left(ac^{5}\right)^{\frac{1}{6}}ac^{2}d+\left(ac^{5}\right)^{\frac{2}{3}}e\right)\log\left(x^{2}-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="giac")
```

```
[Out] -1/6*abs(c)*e*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + d*x/c - 1/3*(a*c^5)^(1
/6)*d*arctan(x/(a/c)^(1/6))/c^2 - 1/6*((a*c^5)^(1/6)*a*c^2*d + sqrt(3)*(a*c
^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/6*
((a*c^5)^(1/6)*a*c^2*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/
c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d - (a*c
^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) + 1/12*
(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d + (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)
^(1/6) + (a/c)^(1/3))/(a*c^4)
```

maple [A] time = 0.08, size = 334, normalized size = 1.07

$$-\frac{\left(\frac{a}{c}\right)^{\frac{7}{6}}\sqrt{3}\,d\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}e\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{dx}{c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}d\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c}+\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^3)/(c+a/x^6),x)
```

```
[Out] 1/c*d*x-1/12*(a/c)^(7/6)/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2
)*d+1/12*(a/c)^(2/3)/a*e*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))-1/6/c*(a
/c)^(1/6)*arctan(2/(a/c)^(1/6)*x+3^(1/2))*d-1/6*(a/c)^(2/3)*3^(1/2)/a*e*arc
tan(2/(a/c)^(1/6)*x+3^(1/2))+1/12*(a/c)^(2/3)/a*e*ln(x^2-3^(1/2)*(a/c)^(1/6
)*x+(a/c)^(1/3))+1/12/c*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(
a/c)^(1/6)*d+1/6*(a/c)^(2/3)*3^(1/2)/a*e*arctan(2/(a/c)^(1/6)*x-3^(1/2))-1/
6/c*(a/c)^(1/6)*arctan(2/(a/c)^(1/6)*x-3^(1/2))*d-1/6*(a/c)^(2/3)/a*e*ln(x^
2+(a/c)^(1/3))-1/3/c*(a/c)^(1/6)*d*arctan(1/(a/c)^(1/6)*x)
```

maxima [A] time = 1.53, size = 295, normalized size = 0.95

$$\frac{dx}{c}-\frac{\frac{2c^{\frac{1}{3}}e\log\left(c^{\frac{1}{3}}x^{2}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{4a^{\frac{1}{3}}d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}+\frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d-a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^{2}+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}}-\frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}d+a^{\frac{2}{3}}ce\right)\log\left(c^{\frac{1}{3}}x^{2}-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}}+\frac{2\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{7}{6}}e+a^{\frac{4}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2c^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}-\frac{2\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{7}{6}}e-a^{\frac{4}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2c^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6),x, algorithm="maxima")
```

```
[Out] d*x/c - 1/12*(2*c^(1/3)*e*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*a^(1/3)*d*
arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*a^
(7/6)*sqrt(c)*d - a^(2/3)*c*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
+ a^(1/3))/(a*c^(2/3)) - (sqrt(3)*a^(7/6)*sqrt(c)*d + a^(2/3)*c*e)*log(c^(1
/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3)*a^(
5/6)*c^(7/6)*e + a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c
^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(sqrt(
3)*a^(5/6)*c^(7/6)*e - a^(4/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3)*a^(
1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/c
```

mupad [B] time = 3.10, size = 1308, normalized size = 4.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^3)/(c + a/x^6),x)
```

```
[Out] log(e*x*(-a^3*c^7)^(1/2) - a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + a^2*c^3*d*
x)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^
3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(
a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)
^(1/2))/(a^2*c^7))^(1/3) - a^2*c^3*d*x)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/
2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + l
og(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) -
 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 3^(1/2)*a
^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*
(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/
2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^
3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) - log(2*e*x*(-a^3*c^7)^(1/2) + a^2*c^4*(
-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^
7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7
)^(1/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i
 + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a*c^5*e^3 + a*d^3*(-a^3*c^7)^(1
/2) - 3*a^2*c^4*d^2*e - 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) -
log(a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d
*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 2*e*x*(-a^3*c^7)^(1/2) + 3^(1/2)*
a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2
*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 + 1
/2)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a
^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(2*e*x*(-a^3*c^7)^(1/2) - a^2*c^4*
(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c
^7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*e^3 - a*d^3*(-a^3*c^
7)^(1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1
i - 2*a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a*c^5*e^3 - a*d^3*(-a^3*c^7)^(
1/2) - 3*a^2*c^4*d^2*e + 3*c*d*e^2*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) +
 (d*x)/c
```

sympy [A] time = 2.98, size = 167, normalized size = 0.54

$$\operatorname{RootSum}\left(46656t^6a^2c^7+t^3\left(-1296a^2c^4d^2e+432ac^5e^3\right)+a^3d^6+3a^2cd^4e^2+3ac^2d^2e^4+c^3e^6,\left(t\mapsto t\log\left(x+\frac{-1296t^4ac^5e-6ta^2cd^4+36tac^2d^2e^2-6tc^3e^4}{a^2d^5-2acd^3e^2-3c^2de^4}\right)\right)\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**3)/(c+a/x**6),x)
```

```
[Out] RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d**2*e + 432*a*c**5*
e**3) + a**3*d**6 + 3*a**2*c*d**4*e**2 + 3*a*c**2*d**2*e**4 + c**3*e**6, La
mbda(_t, _t*log(x + (-1296*_t**4*a*c**5*e - 6*_t*a**2*c*d**4 + 36*_t*a*c**2
*d**2*e**2 - 6*_t*c**3*e**4)/(a**2*d**5 - 2*a*c*d**3*e**2 - 3*c**2*d*e**4))
)) + d*x/c
```

3.39 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=716

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+\right.}{}$$

Rubi [A] time = 1.63, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1393, 1502, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d -
2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e - (b^2*d -
 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d - 2*a*c
*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1393

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{x^3\left(e+dx^3\right)}{a+bx^3+cx^6}\,dx$$

$$=\frac{dx}{c}-\frac{\int \frac{ad+(bd-ce)x^3}{a+bx^3+cx^6}\,dx}{c}$$

$$=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+\ldots}}{2c}$$

$$=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{\ldots}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}\ldots}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4a\ldots}\right.}$$

$$=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\ldots}{3\sqrt[3]{2}\,c^{4/3}\ldots}$$

$$=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\ldots}{3\sqrt[3]{2}\,c^{4/3}\ldots}$$

$$=\frac{dx}{c}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\ldots}{\sqrt[3]{b+\ldots}}}{\ldots}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/\ldots}}$$

<u>**Mathematica**</u> **[C]** time = 0.05, size = 88, normalized size = 0.12

$$\frac{dx}{c}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3bd\log(x-\#1)-\#1^3ce\log(x-\#1)+ad\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*d*Log[x - #1] + b*d*Log[x - #1
]*#1^3 - c*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

<u>**IntegrateAlgebraic**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x^3)/(c + a/x^6 + b/x^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + \frac{e}{x^3}}{c + \frac{b}{x^3} + \frac{a}{x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] integrate((d + e/x^3)/(c + b/x^3 + a/x^6), x)
```

maple [C] time = 0.02, size = 67, normalized size = 0.09

$$\frac{dx}{c} + \frac{\left((-bd + ce)\,\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^3 - ad\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right) + x\right)}{3c\left(2\,\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^5 c + \mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^3)/(c+a/x^6+b/x^3),x)
```

```
[Out] 1/c*d*x+1/3/c*sum(((-b*d+c*e)*_R^3-a*d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=Root
Of(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{dx}{c} + \frac{-\int \frac{(bd-ce)x^3+ad}{cx^6+bx^3+a}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^3)/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate(-((b*d - c*e)*x^3 + a*d)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 29.42, size = 11453, normalized size = 16.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^3)/(c + a/x^6 + b/x^3),x)
```

```
[Out] log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e
^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3
+ 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*
e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^
2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4
```

```
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3
 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*
a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^
2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^
2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6
*a*b*c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c
^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d
^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*
e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*
c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^
3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^
2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3
*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2
+ 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3
*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^
3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e
^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c
^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48
*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*
e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
- 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e +
 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2
*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2
*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e
+ 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^
2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^
2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d
^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b
^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))
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/(c^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b
*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32
*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3
- 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e
^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e
- 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*e*
x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((
b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 -
 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) +
48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c
^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(
1/3))/4)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^
4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e +
 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
 b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^2*d^3
 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6*a*b*
c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^
3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 +
2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4
*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d
^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e + 2
*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c*d^
2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*c^2
*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3
- b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 +
 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d
^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*
d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2
*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))
^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3
*e*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2
*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^
3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*
b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3)
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)^(1/3))/4)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 -
 b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a
^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*
b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*
e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*
c - b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2*c^2*
d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e + 6*a
*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5
*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e
+ 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^
2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b
^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4
*(4*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d^3*e
+ 2*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*b^4*c
*d^2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a*b^2*
c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e
^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^
2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*
c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c
^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*
d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6
)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)*(81*
a*c^3*e*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b
^2)^2*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c
^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48
*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/4)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*
e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3
+ 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e -
 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2
*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3 + a^2
*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d^2*e
+ 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^
2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 - b*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^
2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d
^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b
^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e + 3*b^2*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 - b^5*d
```

```
^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3 + 3*
b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e - 9*a
*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^3 + b^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*
c^4*e^3 - b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2
*d*e^2 + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^5*c*d^3 - 3*b^6*c*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*
b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 - 6*a*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2
*c^3*d^2*e + 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^
2*c^6)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)
*(81*a*c^3*e*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*
c - b^2)^2*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 -
b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^
2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b
^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e
 + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
 - b^2)^3))^(1/3))/4)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2
*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b^3*c^2
*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^6*c*d^
2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 27*a*b^
4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^2*c^2*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(c^4*(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*d^3 - b*c^3*e^3
+ a^2*c^2*d^3 + 3*b^2*c^2*d*e^2 - 3*a*b^2*c*d^3 - 3*a*c^3*d*e^2 - 3*b^3*c*d
^2*e + 6*a*b*c^2*d^2*e))/c)*((b^7*d^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a*b^2*c^4*e^3 + b*c^3*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^5*c^2*d*e^2 + 32*a^2*b
^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*b^
6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d*e^2 + 2
7*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d^2*e - 3*b^
2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*d^4 - 2*a*c^4*e^4 -
b^5*d^3*e + 2*a^3*c^2*d^4 + b^2*c^3*e^4 - 4*a^2*b^2*c*d^4 - 3*b^3*c^2*d*e^3
 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e + 4*a^2*b*c^2*d^3*e
- 9*a*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^3 - b^4*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 16*a^2*c^5*e^3 - b^4*c^3*e^3 - 32*a^3*b*c^3*d^3 + 8*a
*b^2*c^4*e^3 + b*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d^2*e + 3*b^
5*c^2*d*e^2 + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^5*c*d^3 - 3*b^6*c*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^3*c^3*d*e^2 + 27*a*b^4*c^2*d^2*e + 48*a^2*b*c^4*d*e^2 + 6*a*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
2*b^2*c^3*d^2*e - 3*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a
^2*b^2*c^6)))^(1/3) + (d*x)/c
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
```

```
[Out] Timed out
```

3.40 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}} \, dx$

Optimal. Leaf size=753

$$-\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{}\right.}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}$$

Rubi [A] time = 1.44, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1394, 1503, 1415, 1169, 634, 618, 204, 628}

$$-\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}+\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[(
Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8*a
^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*
ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8)
)])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqr
t[c]*e)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]
*a^(1/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqrt[2]*
d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 -
Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[
c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*
Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c
]*e)*Log[a^(1/4) + Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*S
qrt[2*(2 - Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]
*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sq
rt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*
e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqr
t[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1394

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + a/x^(2*n))^p, x] /; FreeQ[{a,
 c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[p, q] && NegQ[n]
```

Rule 1415

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[a/c, 4]}, Dist[1/(2*Sqrt[2]*c*q^3), Int[(Sqrt[2]*d*q - (d - e*q^2)*x
^(n/2))/(q^2 - Sqrt[2]*q*x^(n/2) + x^n), x], x] + Dist[1/(2*Sqrt[2]*c*q^3),
 Int[(Sqrt[2]*d*q + (d - e*q^2)*x^(n/2))/(q^2 + Sqrt[2]*q*x^(n/2) + x^n), x
], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &
& NeQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && PosQ[a*c]
```

Rule 1503

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + c*x^(2*n))^(p +
1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int
[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx = \int \frac{x^4\left(e+dx^4\right)}{a+cx^8}\,dx$$

$$= \frac{dx}{c} - \frac{\int \frac{ad-cex^4}{a+cx^8}\,dx}{c}$$

$$= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\left(-ad-\sqrt{a}\sqrt{c}e\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}c^{5/4}} - \frac{\int \frac{\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}+\left(ad+\sqrt{a}\sqrt{c}e\right)x^2}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x^2}{\sqrt[4]{c}}+x^4}\,dx}{2\sqrt{2}a^{3/4}c^{5/4}}$$

$$= \frac{dx}{c} - \frac{\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{11/8}d}{c^{3/8}}-\left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(ad+\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}c^{7/8}} - \frac{\int \frac{\frac{\sqrt{2\left(2-\sqrt{2}\right)}a^{11/8}d}{c^{3/8}}+\left(\frac{\sqrt{2}a^{5/4}d}{\sqrt[4]{c}}-\frac{\sqrt[4]{a}\left(ad+\sqrt{a}\sqrt{c}e\right)}{\sqrt[4]{c}}\right)x}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}a^{9/8}c^{7/8}} - \cdots$$

$$= \frac{dx}{c} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x^2}\,dx}{8\sqrt{2}\sqrt[4]{a}c^{5/4}} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{c}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}x}{\sqrt[8]{c}}+x \cdots}}{8\sqrt{2}\sqrt[4]{a}c^{5/4}} \cdots$$

$$= \frac{dx}{c} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt[4]{a}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}} + \frac{\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\text{lo}\cdots}{8\sqrt{2}\cdots}$$

$$= \frac{dx}{c} + \frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}} - \frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\cdots}{8a^{3/8}c^{9/8}}$$

Mathematica [A] time = 0.90, size = 551, normalized size = 0.73

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (8*a*c^(5/8)*d*x + 2*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(a^(
5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x
^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(a^(5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c
]*d*Sin[Pi/8]) + 2*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(-(a^(
5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*
x^2 - 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(-(a^(5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sq
rt[c]*d*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*(a
^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*S
ec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*S
in[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(
9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - Log[a^(1/4) + c^(1/4)*x
^2 + 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*
c*e*Sin[Pi/8]))/(8*a*c^(13/8))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e/x^4)/(c + a/x^8), x]
```

fricas [B] time = 2.07, size = 3378, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="fricas")
```

```
[Out] -1/8*(4*c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(
1/4)*arctan(-((3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a
*c^7*e^7 + (a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^
2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)))*sqrt(((a^4
*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8)*x^
2 - (2*a^3*c^7*d*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - a^4*c^2*d^6 + 7*a^3*c^3*d^4*e^2 -
 7*a^2*c^4*d^2*e^4 + a*c^5*e^6)*sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*
e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3
*e + 4*c*d*e^3)/(a*c^4)))/(a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 -
 4*a*c^3*d^2*e^6 + c^4*e^8))*sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2
 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e
+ 4*c*d*e^3)/(a*c^4)) - ((a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*x*sqrt(-(a^4*d^8 -
 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c
^9)) + (3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2*e^5 - a*c^7*e^
7)*x)*sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))*
(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*
d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)/(a^5*
d^10 - 3*a^4*c*d^8*e^2 - 14*a^3*c^2*d^6*e^4 - 14*a^2*c^3*d^4*e^6 - 3*a*c^4*
d^2*e^8 + c^5*e^10)) - 4*c*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d
*e^3)/(a*c^4))^(1/4)*arctan(((3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*
c^6*d^2*e^5 - a*c^7*e^7 - (a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*sqrt(-(a^4*d^8 -
12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^
9)))*sqrt(((a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^
6 + c^4*e^8)*x^2 + (2*a^3*c^7*d*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^
2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^4*c^2*d^6 - 7*a^
3*c^3*d^4*e^2 + 7*a^2*c^4*d^2*e^4 - a*c^5*e^6)*sqrt((a*c^4*sqrt(-(a^4*d^8 -
 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c
^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4)))/(a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2
*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8))*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*
c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4
*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(3/4) + ((a^4*c^8*d^3 - 3*a^3*c^9*d*e^2)*x*s
qrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 +
c^4*e^8)/(a^3*c^9)) - (3*a^4*c^4*d^6*e - 19*a^3*c^5*d^4*e^3 + 9*a^2*c^6*d^2
*e^5 - a*c^7*e^7)*x)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/
(a*c^4))^(3/4))/(a^5*d^10 - 3*a^4*c*d^8*e^2 - 14*a^3*c^2*d^6*e^4 - 14*a^2*c
^3*d^4*e^6 - 3*a*c^4*d^2*e^8 + c^5*e^10)) + c*((a*c^4*sqrt(-(a^4*d^8 - 12*a
^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9))
```

```
+ 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*
a*c^2*d^2*e^4 + c^3*e^6)*x + (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 -
6*a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2
+ 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e -
 4*c*d*e^3)/(a*c^4))^(1/4)) - c*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e -
4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4
+ c^3*e^6)*x - (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^
4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*
e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a
*c^4))^(1/4)) - c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a
*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x
+ (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*
c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a*c^3*d
*e^4)*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)
) + c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)
*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x - (a^2*c^6*e
*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6
+ c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a*c^3*d*e^4)*(-(a*c
^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^
6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)) - 8*d*x)/c
```

giac [A] time = 0.81, size = 647, normalized size = 0.86

$$\frac{dx}{c} - \frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8ac} - \frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8ac} + \frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8ac} + \frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8ac} - \frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16ac} + \frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e + ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16ac} + \frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 + x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16ac} - \frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e - ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2 - x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}} + \left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="giac")
```

```
[Out] d*x/c - 1/8*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/
c)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*
(a/c)^(1/8)))/(a*c) - 1/8*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sq
rt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e
- a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)
^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*sqrt(sqrt(2) + 2)*
(a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt
(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) - 1/16*(c*sqr
t(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2
+ x*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*sqrt(-sqrt
(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*sqrt(sqrt(2) + 2)
*(a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 + x*sqrt(-sqrt
(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) - 1/16*(c*sqrt(sqrt(2) + 2)*(a/c)
^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*log(x^2 - x*sqrt(-sqrt(2) +
2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c)
```

maple [C] time = 0.00, size = 45, normalized size = 0.06

$$\frac{dx}{c} + \frac{\left(\mathrm{RootOf}\left(\_Z^{8}c + a\right)^{4} ce - ad\right) \ln\left(-\mathrm{RootOf}\left(\_Z^{8}c + a\right) + x\right)}{8c^{2}\,\mathrm{RootOf}\left(\_Z^{8}c + a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^4)/(c+a/x^8),x)
```

```
[Out] 1/c*d*x+1/8/c^2*sum((_R^4*c*e-a*d)/_R^7*ln(-_R+x),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{dx}{c}+\frac{-\frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}+\frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{8a}-\frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(c\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e+ad\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}+\frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}-\frac{\left(c\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{5}{8}}e-ad\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{16a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate((c*e*x^4 - a*d)/(c*x^8 + a), x)/c
```

mupad [B] time = 1.22, size = 2520, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^4)/(c + a/x^8),x)
```

```
[Out] (atan((a^3*d^6*x - c^3*e^6*x - a*c^2*d^2*e^4*x + a^2*c*d^4*e^2*x + (2*d*e*x
*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c^6*e*(-(a^
2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3
*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*d^5*(
-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4
*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 2*a^2*c
^2*d^3*e^2*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c
^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/
4) + 3*a*c^3*d*e^4*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) +
 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c
^9))^(1/4)))*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2
*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(
1/4))/4 - (atan((c^3*e^6*x - a^3*d^6*x + a*c^2*d^2*e^4*x - a^2*c*d^4*e^2*x
+ (2*d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6
*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2)))/(a*c^4))/(a^2*c
^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^
3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^
3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d
*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) +
 2*a^2*c^2*d^3*e^2*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) -
4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^
9))^(1/4) + 3*a*c^3*d*e^4*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(
1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/
(a^3*c^9))^(1/4)))*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) -
4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^
9))^(1/4))/4 + atan((c^3*e^6*x*1i - a^3*d^6*x*1i + a*c^2*d^2*e^4*x*1i - a^2
*c*d^4*e^2*x*1i + (d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/
2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))*2i
)/(a*c^4))/(a^2*c^6*e*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2)
 - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3
*c^9))^(5/4) - a^3*c*d^5*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1
/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(
a^3*c^9))^(1/4) + 2*a^2*c^2*d^3*e^2*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-
a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^
9)^(1/2))/(a^3*c^9))^(1/4) + 3*a*c^3*d*e^4*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2
*e^4*(-a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(
-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4)))*((a^2*d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-
a^3*c^9)^(1/2) - 4*a^2*c^6*d*e^3 + 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^
9)^(1/2))/(4096*a^3*c^9))^(1/4)*2i - atan((a^3*d^6*x*1i - c^3*e^6*x*1i - a*
c^2*d^2*e^4*x*1i + a^2*c*d^4*e^2*x*1i + (d*e*x*(a^2*d^4*(-a^3*c^9)^(1/2) +
```

```
c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^
2*(-a^3*c^9)^(1/2))*2i)/(a*c^4))/(a^2*c^6*e*(-(a^2*d^4*(-a^3*c^9)^(1/2) + c
^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2*e^2
*(-a^3*c^9)^(1/2))/(a^3*c^9))^(5/4) - a^3*c*d^5*(-(a^2*d^4*(-a^3*c^9)^(1/2)
 + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e - 6*a*c*d^2
*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 2*a^2*c^2*d^3*e^2*(-(a^2*d^4*(-a^
3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3*e
 - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4) + 3*a*c^3*d*e^4*(-(a^2*
d^4*(-a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c
^5*d^3*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(a^3*c^9))^(1/4)))*(-(a^2*d^4*(-
a^3*c^9)^(1/2) + c^2*e^4*(-a^3*c^9)^(1/2) + 4*a^2*c^6*d*e^3 - 4*a^3*c^5*d^3
*e - 6*a*c*d^2*e^2*(-a^3*c^9)^(1/2))/(4096*a^3*c^9))^(1/4)*2i + (d*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**4)/(c+a/x**8),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=433

$$\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}\right.}{2\sqrt[4]{2}}$$

Rubi [A] time = 0.99, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1393, 1502, 1422, 212, 208, 205}

$$\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e + (b^2*d
- 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
+ ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1393

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(
n_))^(q_.), x_Symbol] :> Int[x^(n*(2*p + q))*(e + d/x^n)^q*(c + b/x^n + a/x
^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && IntegersQ[
p, q] && NegQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\int \frac{x^4\left(e+dx^4\right)}{a+bx^4+cx^8}\,dx$$

$$=\frac{dx}{c}-\frac{\int \frac{ad+(bd-ce)x^4}{a+bx^4+cx^8}\,dx}{c}$$

$$=\frac{dx}{c}-\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}}{2c}$$

$$=\frac{dx}{c}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\int \frac{}{\sqrt{-b+\sqrt{b^2-}}}}{2c\sqrt{-b+\sqrt{b^2-4ac}}}$$

$$=\frac{dx}{c}+\frac{\left(bd-ce+\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(bd-ce-\frac{b^2d-2acd-bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}}{\sqrt[4]{-b+\sqrt{b^2}}}\right.}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.07, size = 88, normalized size = 0.20

$$\frac{dx}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4bd\log(x-\#1)-\#1^4ce\log(x-\#1)+ad\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*d*Log[x - #1] + b*d*Log[x - #1
]*#1^4 - c*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
[Out] IntegrateAlgebraic[(d + e/x^4)/(c + a/x^8 + b/x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Evaluation time: 6.98Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 67, normalized size = 0.15

$$\frac{dx}{c}+\frac{\left((-bd+ce)\,\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^4-ad\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)+x\right)}{4c\left(2\,\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(\_Z^8c+b\_Z^4+a\right)^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e/x^4)/(c+a/x^8+b/x^4),x)
[Out] 1/c*d*x+1/4/c*sum(((-b*d+c*e)*_R^4-a*d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{dx}{c}+\frac{-\int\frac{(bd-ce)x^4+ad}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x^4)/(c+a/x^8+b/x^4),x, algorithm="maxima")
[Out] d*x/c + integrate(-((b*d - c*e)*x^4 + a*d)/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 9.24, size = 50213, normalized size = 115.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e/x^4)/(c + a/x^8 + b/x^4),x)
[Out] atan((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^
2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c
```

- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*
c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^
3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3
*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c
*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*
d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*
b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b
^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2
 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(
-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
+ 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*
d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*
c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e
^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^
2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a
^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 +
 c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16
*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4
*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b

```
^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 + b^4*d
^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e
^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3
*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*
b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a
^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2
)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*
d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8
*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b
^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^
2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(3/4) + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b
*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*
c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^
3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 2
0*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*
c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*
d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*
b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13
*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^
4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) -
 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^
2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c
^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^
4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^
3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d
^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16
*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*
a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e
^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*
(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2
*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c -
b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*
```

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c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^5*b*c^
6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 2
56*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^
4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*d^4 + b^4*d^4*(-(4*a*c -
b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c
^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*
b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*
a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200
*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e -
 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b
^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*
c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240
*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e -
 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*
e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) -
 (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2
*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^
5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4
*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^
4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32
*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^
4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 -
 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 +
48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 -
200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a
*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^
3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*
a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4
*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d
^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^
5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5
*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 + b
^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6
*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3
*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
```

- 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*
b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^
2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*
a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b
^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*
c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240
*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e -
 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*
e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(
16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^
7*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*
b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^
2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*
e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*
c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5
*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*d^5
 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a
^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6
*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22
*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c
^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^
3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b
^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b
^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e +
240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*
e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d
^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2
*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*
a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^
3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d
^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^
3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*
c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4

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*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48
*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 20
0*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)))*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*
e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4
 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^
3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^
3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) -
 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288
*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + ata
n(((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 -
 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*
a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*
d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*
a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5
*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 1
28*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d
^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e
^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e
^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e
+ 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^
5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5
 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3
*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*
c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 -
19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b
^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)
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^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a* b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c

^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c
^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*
b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*
a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200
*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5
*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^
4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*
e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 +
6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*
d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*
e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^
3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a
*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*
c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4
*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*
c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4
*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c
^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 -
2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4
*b^2*c^6*d*e))/c + (16*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*
c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5
*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*
b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e -
288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(163
84*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e
))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c
^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b
^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6
*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2
- 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^
2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3
*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^
2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*
a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*d^5 -
4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*
c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^
2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^
4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*

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d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 +
 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b
^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*
c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240
*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e +
 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*
e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) +
 (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^
3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5
*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*
e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5
*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^
8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*
b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a
^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)))*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4
 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 +
16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3
+ 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c
^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d
*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66
*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^
3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8
*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + 2*atan
((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 -
2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a
^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*d^4
+ b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*
c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*
a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e +
 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80
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*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 1
28*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d
^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e
^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e
^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e
- 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c
*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*
d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*
b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b
^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2
 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(
-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
+ 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*
d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a
^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^
4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4
*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*
c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e -
4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*
c^3*d^3*e^3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^
4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 +
 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3
 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*
c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 6
6*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a
^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + ((((4*x*
(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b
^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*d^4 + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^
3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3
*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
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- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*
b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5
*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^
8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*
b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a
^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*1i - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a
^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*
b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^
2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e
 - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*
b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4
+ b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*
c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*
a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6
 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^
2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e
^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^
2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*
c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^
3))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*
c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^
2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d
^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/(((((4*x*(4096*a^5*
b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2
 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^
3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*d^4 + b^4*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*
c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48
*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 20
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0*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e
- 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a
*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4
*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e
+ 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^
3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3
*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3
/4)*1i + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e
^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^
5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22
*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*
b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*
e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
- 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^
3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3
*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c
^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*
e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^
3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3
*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5
- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 +
128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^
4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2
)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2
*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - ((((4*x*(4096*a^5*b*c^6*d
^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*
a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5
)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4
 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3
*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*
d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
```

) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*
c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b
^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288
*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5
*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^
5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^
3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a
^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c
d^4(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4
*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e
- 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i
- (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^
2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a
^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^
4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d
^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 3
2*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 + b^4*d^4*(-(4*a*
c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a
^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4
- 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 +
 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 -
 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*
a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a
^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6
- 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2
*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c
^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*
d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2
*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4
 + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3
*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120
*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) - 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d^3*e - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*d^4 + b^4*d^4*(-(4*a*c -
b^2)^5)^(1/2) + b^5*c^4*e^4 + c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c
^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*
b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5

)^(1/2) - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*
a*b^6*c^2*d^3*e - 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200
*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + 2*atan((((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2
+ 256*a^3*b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a
^3*b^3*c^6*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^
6*d*e))/c - ((-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 -
c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*
a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8
*e - 256*a^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*
(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4
 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2
*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c -
b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a
^6*c^3*d^5 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5
*d*e^4 + 13*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b
^5*c^2*d^2*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b
^2*c^3*d^3*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e
^4 + 4*a^2*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*
a^4*b^3*c^2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*
c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5
*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*
b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i -
 (4*x*(a^4*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^
3*b^5*d^5*e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5
*c^3*d^4*e^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*
e + 2*a^5*b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^4*e^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5

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*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^
8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*
b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a
^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4) + ((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*
b^5*c^4*d^2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6
*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c
+ ((-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*
e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*
c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a
^2*b^6*c^5*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4
 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3
*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120
*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5
 - 7*a^4*b^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 1
3*a^5*b^2*c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2
*e^3 + 32*a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3
*e^2 + 4*a^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2
*b^6*c*d^3*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^
2*d^4*e))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 -
c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*
a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4
*b^4*d^6 + 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*
e + a^2*b^2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e
^2 + 6*a^2*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3
 - 17*a^4*b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*
b*c^2*d^5*e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e
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^2 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a
*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4
*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e
+ 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^
3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3
*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1
/4))/(((((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^
2 - 2048*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*
d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*
a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5
*e + 3072*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 - b^4*d^4
*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3
 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c
^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^
7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4
*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5
*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^
3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b
^4*c*d^5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*
c^2*d^5 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*
a^3*b^3*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a
^3*b^5*c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3
*e^2 - 19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/
c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*
e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*
c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 +
 2*a^6*c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^
2*c^4*e^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2
```

*b^4*c^2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*
b^2*c^2*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*
e - 4*a^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^
4*b*c^3*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^
4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e
^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d
*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^
2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*
b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*
c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
 - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 2
88*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (
(((4*x*(4096*a^5*b*c^6*d^2 + 4096*a^4*b*c^7*e^2 + 256*a^3*b^5*c^4*d^2 - 204
8*a^4*b^3*c^5*d^2 + 256*a^2*b^5*c^5*e^2 - 2048*a^3*b^3*c^6*e^2 - 16384*a^5*
c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*d^4 - b
^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6
*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3
*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*e - 256*a^2*b^6*c^5*e + 30
72*a^3*b^4*c^6*e - 12288*a^4*b^2*c^7*e)*16i)/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^
4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*
a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4
 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4
 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3
 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3
e(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3
 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6
*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*d^5 - 4*a^6*c^3*d^5 - 7*a^4*b^4*c*d^
5 + 4*a^3*b*c^5*e^5 - a^2*b^7*d^4*e + 12*a^4*c^5*d*e^4 + 13*a^5*b^2*c^2*d^5
 - a^2*b^3*c^4*e^5 + 8*a^5*c^4*d^3*e^2 - 6*a^2*b^5*c^2*d^2*e^3 + 32*a^3*b^3
*c^3*d^2*e^3 - 22*a^3*b^4*c^2*d^3*e^2 + 22*a^4*b^2*c^3*d^3*e^2 + 4*a^3*b^5*
c*d^4*e - 20*a^5*b*c^3*d^4*e + 4*a^2*b^4*c^3*d*e^4 + 4*a^2*b^6*c*d^3*e^2 -
19*a^3*b^2*c^4*d*e^4 - 32*a^4*b*c^4*d^2*e^3 + 5*a^4*b^3*c^2*d^4*e))/c)*(-(b
^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 1
28*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6

```
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*d^6 + 2*a^6*
c^2*d^6 - 2*a^3*c^5*e^6 - 4*a^5*b^2*c*d^6 - 2*a^3*b^5*d^5*e + a^2*b^2*c^4*e
^6 + a^2*b^6*d^4*e^2 - 2*a^4*c^4*d^2*e^4 + 2*a^5*c^3*d^4*e^2 + 6*a^2*b^4*c^
2*d^2*e^4 - 16*a^3*b^2*c^3*d^2*e^4 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^4*e^2 + 10*a^3*b*c^4*d*e^5 + 6*a^4*b^3*c*d^5*e + 2*a^5*b*c^2*d^5*e - 4*a
^2*b^3*c^3*d*e^5 - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^4*e^2 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*d^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 -
 c^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16
*a^2*b*c^6*e^4 + 128*a^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4
*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*d
^4 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + b^5*c^4*e^4 - c^4*e^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a*b^3*c^5*e^4 + 16*a^2*b*c^6*e^4 + 128*a
^3*c^6*d*e^3 - 128*a^4*c^5*d^3*e - 4*b^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 1
20*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e
^2 - 13*a*b^7*c*d^4 - 4*b^8*c*d^3*e + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d*e^3 + 48*a*b^6*c^2*d^3*e + 4*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 4*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128
*a^2*b^2*c^5*d*e^3 - 200*a^2*b^4*c^3*d^3*e - 288*a^3*b*c^5*d^2*e^2 + 320*a^
3*b^2*c^4*d^3*e + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d^3*
e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (d*x)/c
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
[Out] Timed out
```

3.42 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx$

Optimal. Leaf size=62

$$\frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^(1 + n))/(1 + n) + ((c*d + b*e)*x^(1 + 2*n))/(1 + 2*
n) + (c*e*x^(1 + 3*n))/(1 + 3*n)
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx &= \int \left(ad+(bd+ae)x^n+(cd+be)x^{2n}+cex^{3n}\right)\,dx\\ &= adx+\frac{(bd+ae)x^{1+n}}{1+n}+\frac{(cd+be)x^{1+2n}}{1+2n}+\frac{cex^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.15, size = 57, normalized size = 0.92

$$x\left(\frac{x^n(ae+bd)}{n+1}+ad+\frac{x^{2n}(be+cd)}{2n+1}+\frac{cex^{3n}}{3n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(a*d + ((b*d + a*e)*x^n)/(1 + n) + ((c*d + b*e)*x^(2*n))/(1 + 2*n) + (c*e
*x^(3*n))/(1 + 3*n))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] a*d*x + Defer[IntegrateAlgebraic][x^n*(b*d + a*e + c*d*x^n + b*e*x^n + c*e*
x^(2*n)), x]
```

fricas [B] time = 0.94, size = 137, normalized size = 2.21

$$\frac{\left(2\,cen^2+3\,cen+ce\right)xx^{3\,n}+\left(3\,(cd+be)n^2+cd+be+4\,(cd+be)n\right)xx^{2\,n}+\left(6\,(bd+ae)n^2+bd+ae+5\,(bd+ae)n\right)xx^n+\left(6\,adn^3+11\,adn^2+6\,adn+ad\right)x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((2*c*e*n^2 + 3*c*e*n + c*e)*x*x^(3*n) + (3*(c*d + b*e)*n^2 + c*d + b*e + 4
*(c*d + b*e)*n)*x*x^(2*n) + (6*(b*d + a*e)*n^2 + b*d + a*e + 5*(b*d + a*e)*
n)*x*x^n + (6*a*d*n^3 + 11*a*d*n^2 + 6*a*d*n + a*d)*x)/(6*n^3 + 11*n^2 + 6*
n + 1)
```

giac [B] time = 0.35, size = 207, normalized size = 3.34

$$\frac{6\,adn^3x+3\,cdn^2xx^{2\,n}+6\,bdn^2xx^n+2\,cn^2xx^{3\,n}e+3\,bn^2xx^{2\,n}e+6\,an^2xx^ne+11\,adn^2x+4\,cdnxx^{2\,n}+5\,bdnxx^n+3\,cnxx^{3\,n}e+4\,bnxx^{2\,n}e+5\,anxx^ne+6\,adnx+cdxx^{2\,n}+bdxx^n+cxx^{3\,n}e+bxx^{2\,n}e+axx^ne+adx}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (6*a*d*n^3*x + 3*c*d*n^2*x*x^(2*n) + 6*b*d*n^2*x*x^n + 2*c*n^2*x*x^(3*n)*e
+ 3*b*n^2*x*x^(2*n)*e + 6*a*n^2*x*x^n*e + 11*a*d*n^2*x + 4*c*d*n*x*x^(2*n)
+ 5*b*d*n*x*x^n + 3*c*n*x*x^(3*n)*e + 4*b*n*x*x^(2*n)*e + 5*a*n*x*x^n*e + 6
*a*d*n*x + c*d*x*x^(2*n) + b*d*x*x^n + c*x*x^(3*n)*e + b*x*x^(2*n)*e + a*x*
x^n*e + a*d*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

maple [A] time = 0.01, size = 66, normalized size = 1.06

$$\frac{cex\,e^{3n\ln(x)}}{3n+1}+adx+\frac{(ae+bd)\,x\,e^{n\ln(x)}}{n+1}+\frac{(be+cd)\,x\,e^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)*(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] a*d*x+(a*e+b*d)/(n+1)*x*exp(n*ln(x))+(b*e+c*d)/(2*n+1)*x*exp(n*ln(x))^2+c*e
/(3*n+1)*x*exp(n*ln(x))^3
```

maxima [A] time = 0.55, size = 82, normalized size = 1.32

$$adx+\frac{cex^{3\,n+1}}{3\,n+1}+\frac{cdx^{2\,n+1}}{2\,n+1}+\frac{bex^{2\,n+1}}{2\,n+1}+\frac{bdx^{n+1}}{n+1}+\frac{aex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] a*d*x + c*e*x^(3*n + 1)/(3*n + 1) + c*d*x^(2*n + 1)/(2*n + 1) + b*e*x^(2*n
+ 1)/(2*n + 1) + b*d*x^(n + 1)/(n + 1) + a*e*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.66, size = 59, normalized size = 0.95

$$a\,d\,x+\frac{x\,x^{2\,n}\,\left(b\,e+c\,d\right)}{2\,n+1}+\frac{x\,x^n\,\left(a\,e+b\,d\right)}{n+1}+\frac{c\,e\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] a*d*x + (x*x^(2*n)*(b*e + c*d))/(2*n + 1) + (x*x^n*(a*e + b*d))/(n + 1) + (
c*e*x*x^(3*n))/(3*n + 1)
```

sympy [A] time = 1.32, size = 656, normalized size = 10.58

$$\begin{cases} adx + ae\log(x) + bd\log(x) - \frac{be}{x} - \frac{cd}{x} - \frac{ce}{2x^2} & \text{for } n = -1 \\ adx + 2ae\sqrt{x} + 2bd\sqrt{x} + be\log(x) + cd\log(x) - \frac{2ce}{\sqrt{x}} & \text{for } n = -\frac{1}{2} \\ adx + \frac{3aex^{\frac{2}{3}}}{2} + \frac{3bdx^{\frac{2}{3}}}{2} + 3be\sqrt[3]{x} + 3cd\sqrt[3]{x} + ce\log(x) & \text{for } n = -\frac{1}{3} \\ \frac{6adn^3x}{6n^3+11n^2+6n+1} + \frac{11adn^2x}{6n^3+11n^2+6n+1} + \frac{6adnx}{6n^3+11n^2+6n+1} + \frac{adx}{6n^3+11n^2+6n+1} + \frac{6aen^2xx^n}{6n^3+11n^2+6n+1} + \frac{5aenxx^n}{6n^3+11n^2+6n+1} + \frac{aexx^n}{6n^3+11n^2+6n+1} + \frac{6bdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5bdnxx^n}{6n^3+11n^2+6n+1} + \frac{bdxx^n}{6n^3+11n^2+6n+1} + \frac{3ben^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4benxx^{2n}}{6n^3+11n^2+6n+1} + \frac{bexx^{2n}}{6n^3+11n^2+6n+1} + \frac{3cdn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4cdnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{cdxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2cen^2xx^{3n}}{6n^3+11n^2+6n+1} + \frac{3cenxx^{3n}}{6n^3+11n^2+6n+1} + \frac{cexx^{3n}}{6n^3+11n^2+6n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((a*d*x + a*e*log(x) + b*d*log(x) - b*e/x - c*d/x - c*e/(2*x**2),
Eq(n, -1)), (a*d*x + 2*a*e*sqrt(x) + 2*b*d*sqrt(x) + b*e*log(x) + c*d*log(x
) - 2*c*e/sqrt(x), Eq(n, -1/2)), (a*d*x + 3*a*e*x**(2/3)/2 + 3*b*d*x**(2/3)
/2 + 3*b*e*x**(1/3) + 3*c*d*x**(1/3) + c*e*log(x), Eq(n, -1/3)), (6*a*d*n**
3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a*d*n**2*x/(6*n**3 + 11*n**2 + 6*n +
1) + 6*a*d*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*d*x/(6*n**3 + 11*n**2 + 6*n
 + 1) + 6*a*e*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a*e*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + a*e*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*d
*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*d*n*x*x**n/(6*n**3 + 11*n**
2 + 6*n + 1) + b*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*e*n**2*x*x**(2
*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b*e*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6
*n + 1) + b*e*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*d*n**2*x*x**(2*
n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*c*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*
n + 1) + c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*c*e*n**2*x*x**(3*n
)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*e*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n
 + 1) + c*e*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

3.43 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$

Optimal. Leaf size=132

$$a^2dx+\frac{x^{2n+1}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n+1}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{c^2ex^{5n+1}}{5n+1}$$

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1432}

$$a^2dx+\frac{x^{2n+1}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n+1}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{c^2ex^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e
)*x^(1 + 2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(1 + 3*n))/(1 + 3
*n) + (c*(c*d + 2*b*e)*x^(1 + 4*n))/(1 + 4*n) + (c^2*e*x^(1 + 5*n))/(1 + 5*
n)
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx=\int \left(a^2d+a(2bd+ae)x^n+\left(b^2d+2acd+2abe\right)x^{2n}+\left(2bcd+b^2e+2ace\right)\right.$$

$$=a^2dx+\frac{a(2bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d+2acd+2abe\right)x^{1+2n}}{1+2n}+\frac{\left(2bcd+b^2e+2ac\right.}{1+3n}$$

Mathematica [A] time = 0.25, size = 123, normalized size = 0.93

$$x\left(a^2d+\frac{x^{2n}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^n(ae+2bd)}{n+1}+\frac{cx^{4n}(2be+cd)}{4n+1}+\frac{c^2ex^{5n}}{5n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(a^2*d + (a*(2*b*d + a*e)*x^n)/(1 + n) + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^(
2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(3*n))/(1 + 3*n) + (c*(c*d
 + 2*b*e)*x^(4*n))/(1 + 4*n) + (c^2*e*x^(5*n))/(1 + 5*n))
```

IntegrateAlgebraic [F] time = 0.66, size = 0, normalized size = 0.00

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] a^2*d*x + Defer[IntegrateAlgebraic][x^n*(2*a*b*d + a^2*e + b^2*d*x^n + 2*a*
c*d*x^n + 2*a*b*e*x^n + 2*b*c*d*x^(2*n) + b^2*e*x^(2*n) + 2*a*c*e*x^(2*n) +
 c^2*d*x^(3*n) + 2*b*c*e*x^(3*n) + c^2*e*x^(4*n)), x]
```

fricas [B] time = 0.77, size = 495, normalized size = 3.75

$$\frac{\left(24c^2en^4+50c^2en^3+35c^2en^2+10c^2en+c^2e\right)xx^{5n}+\left(30\left(c^2d+2bce\right)n^4+61\left(c^2d+2bce\right)n^3+c^2d+2bce+41\left(c^2d+2bce\right)n^2+11\left(c^2d+2bce\right)n\right)xx^{4n}+\left(40\left(2bcd+\left(b^2+2ac\right)e\right)n^4+78\left(2bcd+\left(b^2+2ac\right)e\right)n^3+2bcd+49\left(2bcd+\left(b^2+2ac\right)e\right)n^2+\left(b^2+2ac\right)e+12\left(2bcd+\left(b^2+2ac\right)e\right)n\right)xx^{3n}+\left(60\left(2abe+\left(b^2+2ac\right)d\right)n^4+107\left(2abe+\left(b^2+2ac\right)d\right)n^3+2abe+59\left(2abe+\left(b^2+2ac\right)d\right)n^2+\left(b^2+2ac\right)d+13\left(2abe+\left(b^2+2ac\right)d\right)n\right)xx^{2n}+\left(120\left(2abd+a^2e\right)n^4+154\left(2abd+a^2e\right)n^3+2abd+a^2e+71\left(2abd+a^2e\right)n^2+14\left(2abd+a^2e\right)n\right)xx^n+\left(120a^2dn^5+274a^2dn^4+225a^2dn^3+85a^2dn^2+15a^2dn+a^2d\right)x}{120n^5+274n^4+225n^3+85n^2+15n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((24*c^2*e*n^4 + 50*c^2*e*n^3 + 35*c^2*e*n^2 + 10*c^2*e*n + c^2*e)*x*x^(5*n
) + (30*(c^2*d + 2*b*c*e)*n^4 + 61*(c^2*d + 2*b*c*e)*n^3 + c^2*d + 2*b*c*e
+ 41*(c^2*d + 2*b*c*e)*n^2 + 11*(c^2*d + 2*b*c*e)*n)*x*x^(4*n) + (40*(2*b*c
*d + (b^2 + 2*a*c)*e)*n^4 + 78*(2*b*c*d + (b^2 + 2*a*c)*e)*n^3 + 2*b*c*d +
49*(2*b*c*d + (b^2 + 2*a*c)*e)*n^2 + (b^2 + 2*a*c)*e + 12*(2*b*c*d + (b^2 +
 2*a*c)*e)*n)*x*x^(3*n) + (60*(2*a*b*e + (b^2 + 2*a*c)*d)*n^4 + 107*(2*a*b*
e + (b^2 + 2*a*c)*d)*n^3 + 2*a*b*e + 59*(2*a*b*e + (b^2 + 2*a*c)*d)*n^2 + (
b^2 + 2*a*c)*d + 13*(2*a*b*e + (b^2 + 2*a*c)*d)*n)*x*x^(2*n) + (120*(2*a*b*
d + a^2*e)*n^4 + 154*(2*a*b*d + a^2*e)*n^3 + 2*a*b*d + a^2*e + 71*(2*a*b*d
+ a^2*e)*n^2 + 14*(2*a*b*d + a^2*e)*n)*x*x^n + (120*a^2*d*n^5 + 274*a^2*d*n
^4 + 225*a^2*d*n^3 + 85*a^2*d*n^2 + 15*a^2*d*n + a^2*d)*x)/(120*n^5 + 274*n
^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

giac [B] time = 0.45, size = 828, normalized size = 6.27

$$\frac{120a^2dn^5x+30c^2dn^4xx^{4n}+80bcdn^4xx^{3n}+60b^2dn^4xx^{2n}+120acdn^4xx^{2n}+240abdn^4xx^n+24c^2n^4xx^{5n}e+60bcn^4xx^{4n}e+40b^2n^4xx^{3n}e+80acn^4xx^{3n}e+120abn^4xx^{2n}e+120a^2n^4xx^ne+274a^2dn^4x+61c^2dn^3xx^{4n}+156bcdn^3xx^{3n}+107b^2dn^3xx^{2n}+214acdn^3xx^{2n}+308abdn^3xx^n+50c^2n^3xx^{5n}e+122bcn^3xx^{4n}e+78b^2n^3xx^{3n}e+156acn^3xx^{3n}e+214abn^3xx^{2n}e+154a^2n^3xx^ne+225a^2dn^3x+41c^2dn^2xx^{4n}+98bcdn^2xx^{3n}+59b^2dn^2xx^{2n}+118acdn^2xx^{2n}+142abdn^2xx^n+35c^2n^2xx^{5n}e+82bcn^2xx^{4n}e+49b^2n^2xx^{3n}e+98acn^2xx^{3n}e+118abn^2xx^{2n}e+71a^2n^2xx^ne+85a^2dn^2x+11c^2dnxx^{4n}+24bcdnxx^{3n}+13b^2dnxx^{2n}+26acdnxx^{2n}+28abdnxx^n+10c^2nxx^{5n}e+22bcnxx^{4n}e+12b^2nxx^{3n}e+24acnxx^{3n}e+26abnxx^{2n}e+14a^2nxx^ne+15a^2dnx+c^2dxx^{4n}+2bcdxx^{3n}+b^2dxx^{2n}+2acdxx^{2n}+2abdxx^n+c^2xx^{5n}e+2bcxx^{4n}e+b^2xx^{3n}e+2acxx^{3n}e+2abxx^{2n}e+a^2xx^ne+a^2dx}{120n^5+274n^4+225n^3+85n^2+15n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (120*a^2*d*n^5*x + 30*c^2*d*n^4*x*x^(4*n) + 80*b*c*d*n^4*x*x^(3*n) + 60*b^2
*d*n^4*x*x^(2*n) + 120*a*c*d*n^4*x*x^(2*n) + 240*a*b*d*n^4*x*x^n + 24*c^2*n
^4*x*x^(5*n)*e + 60*b*c*n^4*x*x^(4*n)*e + 40*b^2*n^4*x*x^(3*n)*e + 80*a*c*n
^4*x*x^(3*n)*e + 120*a*b*n^4*x*x^(2*n)*e + 120*a^2*n^4*x*x^n*e + 274*a^2*d*
n^4*x + 61*c^2*d*n^3*x*x^(4*n) + 156*b*c*d*n^3*x*x^(3*n) + 107*b^2*d*n^3*x*
x^(2*n) + 214*a*c*d*n^3*x*x^(2*n) + 308*a*b*d*n^3*x*x^n + 50*c^2*n^3*x*x^(5
*n)*e + 122*b*c*n^3*x*x^(4*n)*e + 78*b^2*n^3*x*x^(3*n)*e + 156*a*c*n^3*x*x^
(3*n)*e + 214*a*b*n^3*x*x^(2*n)*e + 154*a^2*n^3*x*x^n*e + 225*a^2*d*n^3*x +
 41*c^2*d*n^2*x*x^(4*n) + 98*b*c*d*n^2*x*x^(3*n) + 59*b^2*d*n^2*x*x^(2*n) +
 118*a*c*d*n^2*x*x^(2*n) + 142*a*b*d*n^2*x*x^n + 35*c^2*n^2*x*x^(5*n)*e + 8
2*b*c*n^2*x*x^(4*n)*e + 49*b^2*n^2*x*x^(3*n)*e + 98*a*c*n^2*x*x^(3*n)*e + 1
18*a*b*n^2*x*x^(2*n)*e + 71*a^2*n^2*x*x^n*e + 85*a^2*d*n^2*x + 11*c^2*d*n*x
*x^(4*n) + 24*b*c*d*n*x*x^(3*n) + 13*b^2*d*n*x*x^(2*n) + 26*a*c*d*n*x*x^(2*
n) + 28*a*b*d*n*x*x^n + 10*c^2*n*x*x^(5*n)*e + 22*b*c*n*x*x^(4*n)*e + 12*b^
2*n*x*x^(3*n)*e + 24*a*c*n*x*x^(3*n)*e + 26*a*b*n*x*x^(2*n)*e + 14*a^2*n*x*
x^n*e + 15*a^2*d*n*x + c^2*d*x*x^(4*n) + 2*b*c*d*x*x^(3*n) + b^2*d*x*x^(2*n
) + 2*a*c*d*x*x^(2*n) + 2*a*b*d*x*x^n + c^2*x*x^(5*n)*e + 2*b*c*x*x^(4*n)*e
 + b^2*x*x^(3*n)*e + 2*a*c*x*x^(3*n)*e + 2*a*b*x*x^(2*n)*e + a^2*x*x^n*e +
a^2*d*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

maple [A] time = 0.02, size = 138, normalized size = 1.05

$$\frac{c^2ex\,e^{5n\ln(x)}}{5n+1}+a^2dx+\frac{(ae+2bd)\,ax\,e^{n\ln(x)}}{n+1}+\frac{(2be+cd)\,cx\,e^{4n\ln(x)}}{4n+1}+\frac{\left(2abe+2acd+b^2d\right)x\,e^{2n\ln(x)}}{2n+1}+\frac{\left(2ace+b^2e+2bcd\right)x\,e^{3n\ln(x)}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)*(b*x^n+c*x^(2*n)+a)^2,x)
```

```
[Out] a^2*d*x+(2*a*c*e+b^2*e+2*b*c*d)/(3*n+1)*x*exp(n*ln(x))^3+(2*a*b*e+2*a*c*d+b
^2*d)/(2*n+1)*x*exp(n*ln(x))^2+a*(a*e+2*b*d)/(n+1)*x*exp(n*ln(x))+c*(2*b*e+
c*d)/(1+4*n)*x*exp(n*ln(x))^4+e*c^2/(1+5*n)*x*exp(n*ln(x))^5
```

maxima [A] time = 0.70, size = 208, normalized size = 1.58

$$a^2dx + \frac{c^2ex^{5n+1}}{5n+1} + \frac{c^2dx^{4n+1}}{4n+1} + \frac{2\,bcex^{4n+1}}{4n+1} + \frac{2\,bcdx^{3n+1}}{3n+1} + \frac{b^2ex^{3n+1}}{3n+1} + \frac{2\,acex^{3n+1}}{3n+1} + \frac{b^2dx^{2n+1}}{2n+1} + \frac{2\,acdx^{2n+1}}{2n+1} + \frac{2\,abex^{2n+1}}{2n+1} + \frac{2\,abdx^{n+1}}{n+1} + \frac{a^2ex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] a^2*d*x + c^2*e*x^(5*n + 1)/(5*n + 1) + c^2*d*x^(4*n + 1)/(4*n + 1) + 2*b*c
*e*x^(4*n + 1)/(4*n + 1) + 2*b*c*d*x^(3*n + 1)/(3*n + 1) + b^2*e*x^(3*n + 1
)/(3*n + 1) + 2*a*c*e*x^(3*n + 1)/(3*n + 1) + b^2*d*x^(2*n + 1)/(2*n + 1) +
 2*a*c*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*e*x^(2*n + 1)/(2*n + 1) + 2*a*b*d*x^
(n + 1)/(n + 1) + a^2*e*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.71, size = 131, normalized size = 0.99

$$a^2\,d\,x + \frac{x\,x^{4\,n}\,\left(d\,c^2 + 2\,b\,e\,c\right)}{4\,n+1} + \frac{x\,x^n\,\left(e\,a^2 + 2\,b\,d\,a\right)}{n+1} + \frac{x\,x^{2\,n}\,\left(d\,b^2 + 2\,a\,e\,b + 2\,a\,c\,d\right)}{2\,n+1} + \frac{x\,x^{3\,n}\,\left(e\,b^2 + 2\,c\,d\,b + 2\,a\,c\,e\right)}{3\,n+1} + \frac{c^2\,e\,x\,x^{5\,n}}{5\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] a^2*d*x + (x*x^(4*n)*(c^2*d + 2*b*c*e))/(4*n + 1) + (x*x^n*(a^2*e + 2*a*b*d
))/(n + 1) + (x*x^(2*n)*(b^2*d + 2*a*b*e + 2*a*c*d))/(2*n + 1) + (x*x^(3*n)
*(b^2*e + 2*a*c*e + 2*b*c*d))/(3*n + 1) + (c^2*e*x*x^(5*n))/(5*n + 1)
```

sympy [A] time = 10.97, size = 3128, normalized size = 23.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*log(x) + 2*a*b*d*log(x) - 2*a*b*e/x - 2*a*c*d/
x - a*c*e/x**2 - b**2*d/x - b**2*e/(2*x**2) - b*c*d/x**2 - 2*b*c*e/(3*x**3)
 - c**2*d/(3*x**3) - c**2*e/(4*x**4), Eq(n, -1)), (a**2*d*x + 2*a**2*e*sqrt
(x) + 4*a*b*d*sqrt(x) + 2*a*b*e*log(x) + 2*a*c*d*log(x) - 4*a*c*e/sqrt(x) +
 b**2*d*log(x) - 2*b**2*e/sqrt(x) - 4*b*c*d/sqrt(x) - 2*b*c*e/x - c**2*d/x
- 2*c**2*e/(3*x**(3/2)), Eq(n, -1/2)), (a**2*d*x + 3*a**2*e*x**(2/3)/2 + 3*
a*b*d*x**(2/3) + 6*a*b*e*x**(1/3) + 6*a*c*d*x**(1/3) + 2*a*c*e*log(x) + 3*b
**2*d*x**(1/3) + b**2*e*log(x) + 2*b*c*d*log(x) - 6*b*c*e/x**(1/3) - 3*c**2
*d/x**(1/3) - 3*c**2*e/(2*x**(2/3)), Eq(n, -1/3)), (a**2*d*x + 4*a**2*e*x**
(3/4)/3 + 8*a*b*d*x**(3/4)/3 + 4*a*b*e*sqrt(x) + 4*a*c*d*sqrt(x) + 8*a*c*e*
x**(1/4) + 2*b**2*d*sqrt(x) + 4*b**2*e*x**(1/4) + 8*b*c*d*x**(1/4) + 2*b*c*
e*log(x) + c**2*d*log(x) - 4*c**2*e/x**(1/4), Eq(n, -1/4)), (a**2*d*x + 5*a
**2*e*x**(4/5)/4 + 5*a*b*d*x**(4/5)/2 + 10*a*b*e*x**(3/5)/3 + 10*a*c*d*x**(
3/5)/3 + 5*a*c*e*x**(2/5) + 5*b**2*d*x**(3/5)/3 + 5*b**2*e*x**(2/5)/2 + 5*b
*c*d*x**(2/5) + 10*b*c*e*x**(1/5) + 5*c**2*d*x**(1/5) + c**2*e*log(x), Eq(n
, -1/5)), (120*a**2*d*n**5*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 274*a**2*d*n**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*
n + 1) + 225*a**2*d*n**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 85*a**2*d*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 15*a**2*d*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 a**2*d*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 120*a**2*
e*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 154*a
**2*e*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 7
1*a**2*e*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ 14*a**2*e*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
```

```
+ a**2*e*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240
*a*b*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
308*a*b*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 142*a*b*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 28*a*b*d*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 2*a*b*d*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 120*a*b*e*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 214*a*b*e*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 118*a*b*e*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 26*a*b*e*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 2*a*b*e*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 120*a*c*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 214*a*c*d*n**3*x*x**(2*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 118*a*c*d*n**2*x*x**(2*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 26*a*c*d*n*x*x**(2*n)/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*c*d*x*x**(2*n)/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*a*c*e*n**4*x*x**(3*n)/(
120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*a*c*e*n**3*x*x**
(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*a*c*e*n**2
*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*a*c*
e*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*
c*e*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b
**2*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 107*b**2*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 59*b**2*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 13*b**2*d*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + b**2*d*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 40*b**2*e*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 78*b**2*e*n**3*x*x**(3*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 49*b**2*e*n**2*x*x**(3*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12*b**2*e*n*x*x**(3*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + b**2*e*x*x**(3*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 80*b*c*d*n**4*x*x**(3*n)/(120*
n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*b*c*d*n**3*x*x**(3*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*b*c*d*n**2*x*x
**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*c*d*n*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*b*c*d*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b*c*e
*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12
2*b*c*e*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 82*b*c*e*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 22*b*c*e*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 2*b*c*e*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 30*c**2*d*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 61*c**2*d*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 41*c**2*d*n**2*x*x**(4*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 11*c**2*d*n*x*x**(4*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*d*x*x**(4*n)/(120*n**5 + 274*n*
*4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*c**2*e*n**4*x*x**(5*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*c**2*e*n**3*x*x**(5*n)/(120
*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 35*c**2*e*n**2*x*x**(5*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 10*c**2*e*n*x*x*
*(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*e*x*x**
(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1), True))
```

3.44 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal. Leaf size=218

$$a^3dx+\frac{x^{3n+1}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+\frac{3ax^{2n+1}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n+1}\left(ace+b^2\right.}{5n+1}$$

Rubi [A] time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1432}

$$\frac{x^{3n+1}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+a^3dx+\frac{x^{4n+1}\left(6abce+3ac^2d+3b^2cd+b^3e\right)}{4n+1}+\frac{3ax^{2n+1}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n+1}\left(ace+b^2e+bcd\right)}{5n+1}+\frac{c^2x^{6n+1}(3be+cd)}{6n+1}+\frac{c^3ex^{7n+1}}{7n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a*c*d + a*b
*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x
^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(1 +
 4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c
^2*(c*d + 3*b*e)*x^(1 + 6*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

Rule 1432

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx &= \int \left(a^3d+a^2(3bd+ae)x^n+3a\left(b^2d+acd+abe\right)x^{2n}+\left(b^3d+6abcd+3ab\right.\right.\\ &= a^3dx+\frac{a^2(3bd+ae)x^{1+n}}{1+n}+\frac{3a\left(b^2d+acd+abe\right)x^{1+2n}}{1+2n}+\frac{\left(b^3d+6abcd+\right.}{1}\end{aligned}$$

Mathematica [A] time = 0.43, size = 205, normalized size = 0.94

$$x\left(a^3d+\frac{x^{3n}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^n(ae+3bd)}{n+1}+\frac{3ax^{2n}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n}\left(ace+b^2e+bcd\right)}{5n+1}+\frac{x^{4n}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{4n+1}+\frac{c^2x^{6n}(3be+cd)}{6n+1}+\frac{c^3ex^{7n}}{7n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(a^3*d + (a^2*(3*b*d + a*e)*x^n)/(1 + n) + (3*a*(b^2*d + a*c*d + a*b*e)*x
^(2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^(3*n))/(
1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^(4*n))/(1 + 4*n)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^(5*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*x^(6
*n))/(1 + 6*n) + (c^3*e*x^(7*n))/(1 + 7*n))
```

IntegrateAlgebraic [F] time = 3.40, size = 0, normalized size = 0.00

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] a^3*d*x + Defer[IntegrateAlgebraic][3*a^2*b*d*x^n + a^3*e*x^n + 3*a*b^2*d*x
^(2*n) + 3*a^2*c*d*x^(2*n) + 3*a^2*b*e*x^(2*n) + b^3*d*x^(3*n) + 6*a*b*c*d*
x^(3*n) + 3*a*b^2*e*x^(3*n) + 3*a^2*c*e*x^(3*n) + 3*b^2*c*d*x^(4*n) + 3*a*c
^2*d*x^(4*n) + b^3*e*x^(4*n) + 6*a*b*c*e*x^(4*n) + 3*b*c^2*d*x^(5*n) + 3*b^
2*c*e*x^(5*n) + 3*a*c^2*e*x^(5*n) + c^3*d*x^(6*n) + 3*b*c^2*e*x^(6*n) + c^3
*e*x^(7*n), x]
```

fricas [B] time = 0.88, size = 1209, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((720*c^3*e*n^6 + 1764*c^3*e*n^5 + 1624*c^3*e*n^4 + 735*c^3*e*n^3 + 175*c^3
*e*n^2 + 21*c^3*e*n + c^3*e)*x*x^(7*n) + (840*(c^3*d + 3*b*c^2*e)*n^6 + 203
8*(c^3*d + 3*b*c^2*e)*n^5 + 1849*(c^3*d + 3*b*c^2*e)*n^4 + c^3*d + 3*b*c^2*
e + 820*(c^3*d + 3*b*c^2*e)*n^3 + 190*(c^3*d + 3*b*c^2*e)*n^2 + 22*(c^3*d +
 3*b*c^2*e)*n)*x*x^(6*n) + 3*(1008*(b*c^2*d + (b^2*c + a*c^2)*e)*n^6 + 2412
*(b*c^2*d + (b^2*c + a*c^2)*e)*n^5 + 2144*(b*c^2*d + (b^2*c + a*c^2)*e)*n^4
 + b*c^2*d + 925*(b*c^2*d + (b^2*c + a*c^2)*e)*n^3 + 207*(b*c^2*d + (b^2*c
+ a*c^2)*e)*n^2 + (b^2*c + a*c^2)*e + 23*(b*c^2*d + (b^2*c + a*c^2)*e)*n)*x
*x^(5*n) + (1260*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^6 + 2952*(3*(b
^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^5 + 2545*(3*(b^2*c + a*c^2)*d + (b^3
 + 6*a*b*c)*e)*n^4 + 1056*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^3 + 2
26*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^2 + 3*(b^2*c + a*c^2)*d + (b
^3 + 6*a*b*c)*e + 24*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n)*x*x^(4*n)
 + (1680*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^6 + 3796*((b^3 + 6*a*b
*c)*d + 3*(a*b^2 + a^2*c)*e)*n^5 + 3112*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2
*c)*e)*n^4 + 1219*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^3 + 247*((b^3
 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^2 + (b^3 + 6*a*b*c)*d + 3*(a*b^2 + a
^2*c)*e + 25*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n)*x*x^(3*n) + 3*(25
20*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^6 + 5274*(a^2*b*e + (a*b^2 + a^2*c)*d)*n
^5 + 3929*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^4 + a^2*b*e + 1420*(a^2*b*e + (a*
b^2 + a^2*c)*d)*n^3 + 270*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^2 + (a*b^2 + a^2*
c)*d + 26*(a^2*b*e + (a*b^2 + a^2*c)*d)*n)*x*x^(2*n) + (5040*(3*a^2*b*d + a
^3*e)*n^6 + 8028*(3*a^2*b*d + a^3*e)*n^5 + 5104*(3*a^2*b*d + a^3*e)*n^4 + 3
*a^2*b*d + a^3*e + 1665*(3*a^2*b*d + a^3*e)*n^3 + 295*(3*a^2*b*d + a^3*e)*n
^2 + 27*(3*a^2*b*d + a^3*e)*n)*x*x^n + (5040*a^3*d*n^7 + 13068*a^3*d*n^6 +
13132*a^3*d*n^5 + 6769*a^3*d*n^4 + 1960*a^3*d*n^3 + 322*a^3*d*n^2 + 28*a^3*
d*n + a^3*d)*x)/(5040*n^7 + 13068*n^6 + 13132*n^5 + 6769*n^4 + 1960*n^3 + 3
22*n^2 + 28*n + 1)
```

giac [B] time = 0.78, size = 2134, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] (5040*a^3*d*n^7*x + 840*c^3*d*n^6*x*x^(6*n) + 3024*b*c^2*d*n^6*x*x^(5*n) +
3780*b^2*c*d*n^6*x*x^(4*n) + 3780*a*c^2*d*n^6*x*x^(4*n) + 1680*b^3*d*n^6*x*
x^(3*n) + 10080*a*b*c*d*n^6*x*x^(3*n) + 7560*a*b^2*d*n^6*x*x^(2*n) + 7560*a
^2*c*d*n^6*x*x^(2*n) + 15120*a^2*b*d*n^6*x*x^n + 720*c^3*n^6*x*x^(7*n)*e +
2520*b*c^2*n^6*x*x^(6*n)*e + 3024*b^2*c*n^6*x*x^(5*n)*e + 3024*a*c^2*n^6*x*
x^(5*n)*e + 1260*b^3*n^6*x*x^(4*n)*e + 7560*a*b*c*n^6*x*x^(4*n)*e + 5040*a*
b^2*n^6*x*x^(3*n)*e + 5040*a^2*c*n^6*x*x^(3*n)*e + 7560*a^2*b*n^6*x*x^(2*n)
*e + 5040*a^3*n^6*x*x^n*e + 13068*a^3*d*n^6*x + 2038*c^3*d*n^5*x*x^(6*n) +
7236*b*c^2*d*n^5*x*x^(5*n) + 8856*b^2*c*d*n^5*x*x^(4*n) + 8856*a*c^2*d*n^5*
x*x^(4*n) + 3796*b^3*d*n^5*x*x^(3*n) + 22776*a*b*c*d*n^5*x*x^(3*n) + 15822*
```

```
a*b^2*d*n^5*x*x^(2*n) + 15822*a^2*c*d*n^5*x*x^(2*n) + 24084*a^2*b*d*n^5*x*x
^n + 1764*c^3*n^5*x*x^(7*n)*e + 6114*b*c^2*n^5*x*x^(6*n)*e + 7236*b^2*c*n^5
*x*x^(5*n)*e + 7236*a*c^2*n^5*x*x^(5*n)*e + 2952*b^3*n^5*x*x^(4*n)*e + 1771
2*a*b*c*n^5*x*x^(4*n)*e + 11388*a*b^2*n^5*x*x^(3*n)*e + 11388*a^2*c*n^5*x*x
^(3*n)*e + 15822*a^2*b*n^5*x*x^(2*n)*e + 8028*a^3*n^5*x*x^n*e + 13132*a^3*d
*n^5*x + 1849*c^3*d*n^4*x*x^(6*n) + 6432*b*c^2*d*n^4*x*x^(5*n) + 7635*b^2*c
*d*n^4*x*x^(4*n) + 7635*a*c^2*d*n^4*x*x^(4*n) + 3112*b^3*d*n^4*x*x^(3*n) +
18672*a*b*c*d*n^4*x*x^(3*n) + 11787*a*b^2*d*n^4*x*x^(2*n) + 11787*a^2*c*d*n
^4*x*x^(2*n) + 15312*a^2*b*d*n^4*x*x^n + 1624*c^3*n^4*x*x^(7*n)*e + 5547*b*
c^2*n^4*x*x^(6*n)*e + 6432*b^2*c*n^4*x*x^(5*n)*e + 6432*a*c^2*n^4*x*x^(5*n)
*e + 2545*b^3*n^4*x*x^(4*n)*e + 15270*a*b*c*n^4*x*x^(4*n)*e + 9336*a*b^2*n^
4*x*x^(3*n)*e + 9336*a^2*c*n^4*x*x^(3*n)*e + 11787*a^2*b*n^4*x*x^(2*n)*e +
5104*a^3*n^4*x*x^n*e + 6769*a^3*d*n^4*x + 820*c^3*d*n^3*x*x^(6*n) + 2775*b*
c^2*d*n^3*x*x^(5*n) + 3168*b^2*c*d*n^3*x*x^(4*n) + 3168*a*c^2*d*n^3*x*x^(4*
n) + 1219*b^3*d*n^3*x*x^(3*n) + 7314*a*b*c*d*n^3*x*x^(3*n) + 4260*a*b^2*d*n
^3*x*x^(2*n) + 4260*a^2*c*d*n^3*x*x^(2*n) + 4995*a^2*b*d*n^3*x*x^n + 735*c^
3*n^3*x*x^(7*n)*e + 2460*b*c^2*n^3*x*x^(6*n)*e + 2775*b^2*c*n^3*x*x^(5*n)*e
 + 2775*a*c^2*n^3*x*x^(5*n)*e + 1056*b^3*n^3*x*x^(4*n)*e + 6336*a*b*c*n^3*x
*x^(4*n)*e + 3657*a*b^2*n^3*x*x^(3*n)*e + 3657*a^2*c*n^3*x*x^(3*n)*e + 4260
*a^2*b*n^3*x*x^(2*n)*e + 1665*a^3*n^3*x*x^n*e + 1960*a^3*d*n^3*x + 190*c^3*
d*n^2*x*x^(6*n) + 621*b*c^2*d*n^2*x*x^(5*n) + 678*b^2*c*d*n^2*x*x^(4*n) + 6
78*a*c^2*d*n^2*x*x^(4*n) + 247*b^3*d*n^2*x*x^(3*n) + 1482*a*b*c*d*n^2*x*x^(
3*n) + 810*a*b^2*d*n^2*x*x^(2*n) + 810*a^2*c*d*n^2*x*x^(2*n) + 885*a^2*b*d*
n^2*x*x^n + 175*c^3*n^2*x*x^(7*n)*e + 570*b*c^2*n^2*x*x^(6*n)*e + 621*b^2*c
*n^2*x*x^(5*n)*e + 621*a*c^2*n^2*x*x^(5*n)*e + 226*b^3*n^2*x*x^(4*n)*e + 13
56*a*b*c*n^2*x*x^(4*n)*e + 741*a*b^2*n^2*x*x^(3*n)*e + 741*a^2*c*n^2*x*x^(3
*n)*e + 810*a^2*b*n^2*x*x^(2*n)*e + 295*a^3*n^2*x*x^n*e + 322*a^3*d*n^2*x +
 22*c^3*d*n*x*x^(6*n) + 69*b*c^2*d*n*x*x^(5*n) + 72*b^2*c*d*n*x*x^(4*n) + 7
2*a*c^2*d*n*x*x^(4*n) + 25*b^3*d*n*x*x^(3*n) + 150*a*b*c*d*n*x*x^(3*n) + 78
*a*b^2*d*n*x*x^(2*n) + 78*a^2*c*d*n*x*x^(2*n) + 81*a^2*b*d*n*x*x^n + 21*c^3
*n*x*x^(7*n)*e + 66*b*c^2*n*x*x^(6*n)*e + 69*b^2*c*n*x*x^(5*n)*e + 69*a*c^2
*n*x*x^(5*n)*e + 24*b^3*n*x*x^(4*n)*e + 144*a*b*c*n*x*x^(4*n)*e + 75*a*b^2*
n*x*x^(3*n)*e + 75*a^2*c*n*x*x^(3*n)*e + 78*a^2*b*n*x*x^(2*n)*e + 27*a^3*n*
x*x^n*e + 28*a^3*d*n*x + c^3*d*x*x^(6*n) + 3*b*c^2*d*x*x^(5*n) + 3*b^2*c*d*
x*x^(4*n) + 3*a*c^2*d*x*x^(4*n) + b^3*d*x*x^(3*n) + 6*a*b*c*d*x*x^(3*n) + 3
*a*b^2*d*x*x^(2*n) + 3*a^2*c*d*x*x^(2*n) + 3*a^2*b*d*x*x^n + c^3*x*x^(7*n)*
e + 3*b*c^2*x*x^(6*n)*e + 3*b^2*c*x*x^(5*n)*e + 3*a*c^2*x*x^(5*n)*e + b^3*x
*x^(4*n)*e + 6*a*b*c*x*x^(4*n)*e + 3*a*b^2*x*x^(3*n)*e + 3*a^2*c*x*x^(3*n)*
e + 3*a^2*b*x*x^(2*n)*e + a^3*x*x^n*e + a^3*d*x)/(5040*n^7 + 13068*n^6 + 13
132*n^5 + 6769*n^4 + 1960*n^3 + 322*n^2 + 28*n + 1)
```

maple [A] time = 0.02, size = 226, normalized size = 1.04

$$\frac{c^{3}ex\,e^{7n\ln(x)}}{7n+1}+a^{3}dx+\frac{(ae+3bd)\,a^{2}x\,e^{n\ln(x)}}{n+1}+\frac{(3be+cd)\,c^{2}x\,e^{6n\ln(x)}}{6n+1}+\frac{3\left(abe+acd+b^{2}d\right)ax\,e^{2n\ln(x)}}{2n+1}+\frac{3\left(ace+b^{2}e+bcd\right)cx\,e^{5n\ln(x)}}{5n+1}+\frac{\left(3a^{2}ce+3a\,b^{2}e+6abcd+b^{3}d\right)x\,e^{3n\ln(x)}}{3n+1}+\frac{\left(6abce+3a\,c^{2}d+b^{3}e+3b^{2}cd\right)x\,e^{4n\ln(x)}}{4n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)*(b*x^n+c*x^(2*n)+a)^3,x)
```

```
[Out] a^3*d*x+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)/(4*n+1)*x*exp(n*ln(x))^4+(3*a
^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)/(3*n+1)*x*exp(n*ln(x))^3+a^2*(a*e+3*b*d)/
(n+1)*x*exp(n*ln(x))+c^2*(3*b*e+c*d)/(1+6*n)*x*exp(n*ln(x))^6+c^3*e/(1+7*n)
*x*exp(n*ln(x))^7+3*a*(a*b*e+a*c*d+b^2*d)/(2*n+1)*x*exp(n*ln(x))^2+3*c*(a*c
*e+b^2*e+b*c*d)/(5*n+1)*x*exp(n*ln(x))^5
```

maxima [A] time = 0.88, size = 386, normalized size = 1.77

$$a^{3}dx+\frac{c^{3}ex^{7n+1}}{7n+1}+\frac{c^{3}dx^{6n+1}}{6n+1}+\frac{3\,bc^{2}ex^{6n+1}}{6n+1}+\frac{3\,bc^{2}dx^{5n+1}}{5n+1}+\frac{3\,b^{2}cex^{5n+1}}{5n+1}+\frac{3\,ac^{2}ex^{5n+1}}{5n+1}+\frac{3\,b^{2}cdx^{4n+1}}{4n+1}+\frac{3\,ac^{2}dx^{4n+1}}{4n+1}+\frac{b^{3}ex^{4n+1}}{4n+1}+\frac{6\,abcex^{4n+1}}{4n+1}+\frac{b^{3}dx^{3n+1}}{3n+1}+\frac{6\,abcdx^{3n+1}}{3n+1}+\frac{3\,ab^{2}ex^{3n+1}}{3n+1}+\frac{3\,a^{2}cex^{3n+1}}{3n+1}+\frac{3\,ab^{2}dx^{2n+1}}{2n+1}+\frac{3\,a^{2}cdx^{2n+1}}{2n+1}+\frac{3\,a^{2}bex^{2n+1}}{2n+1}+\frac{3\,a^{2}bdx^{n+1}}{n+1}+\frac{a^{3}ex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] a^3*d*x + c^3*e*x^(7*n + 1)/(7*n + 1) + c^3*d*x^(6*n + 1)/(6*n + 1) + 3*b*c
^2*e*x^(6*n + 1)/(6*n + 1) + 3*b*c^2*d*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*e*x^
(5*n + 1)/(5*n + 1) + 3*a*c^2*e*x^(5*n + 1)/(5*n + 1) + 3*b^2*c*d*x^(4*n +
1)/(4*n + 1) + 3*a*c^2*d*x^(4*n + 1)/(4*n + 1) + b^3*e*x^(4*n + 1)/(4*n + 1
) + 6*a*b*c*e*x^(4*n + 1)/(4*n + 1) + b^3*d*x^(3*n + 1)/(3*n + 1) + 6*a*b*c
*d*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*e*x^(3*n + 1)/(3*n + 1) + 3*a^2*c*e*x^(3
*n + 1)/(3*n + 1) + 3*a*b^2*d*x^(2*n + 1)/(2*n + 1) + 3*a^2*c*d*x^(2*n + 1)
/(2*n + 1) + 3*a^2*b*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*d*x^(n + 1)/(n + 1)
+ a^3*e*x^(n + 1)/(n + 1)
```

mupad [**B**]   time = 1.85, size = 227, normalized size = 1.04

$$a^3\,d\,x+\frac{x\,x^n\,\left(e\,a^3+3\,b\,d\,a^2\right)}{n+1}+\frac{x\,x^{2\,n}\,\left(3\,e\,a^2\,b+3\,c\,d\,a^2+3\,d\,a\,b^2\right)}{2\,n+1}+\frac{x\,x^{5\,n}\,\left(3\,e\,b^2\,c+3\,d\,b\,c^2+3\,a\,e\,c^2\right)}{5\,n+1}+\frac{x\,x^{3\,n}\,\left(3\,c\,e\,a^2+3\,e\,a\,b^2+6\,c\,d\,a\,b+d\,b^3\right)}{3\,n+1}+\frac{x\,x^{4\,n}\,\left(e\,b^3+3\,d\,b^2\,c+6\,a\,e\,b\,c+3\,a\,d\,c^2\right)}{4\,n+1}+\frac{x\,x^{6\,n}\,\left(d\,c^3+3\,b\,e\,c^2\right)}{6\,n+1}+\frac{c^3\,e\,x\,x^{7\,n}}{7\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] a^3*d*x + (x*x^n*(a^3*e + 3*a^2*b*d))/(n + 1) + (x*x^(2*n)*(3*a*b^2*d + 3*a
^2*b*e + 3*a^2*c*d))/(2*n + 1) + (x*x^(5*n)*(3*a*c^2*e + 3*b*c^2*d + 3*b^2*
c*e))/(5*n + 1) + (x*x^(3*n)*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d))/(
3*n + 1) + (x*x^(4*n)*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e))/(4*n + 1
) + (x*x^(6*n)*(c^3*d + 3*b*c^2*e))/(6*n + 1) + (c^3*e*x*x^(7*n))/(7*n + 1)
```

sympy [**A**]   time = 89.55, size = 9190, normalized size = 42.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Piecewise((a**3*d*x + a**3*e*log(x) + 3*a**2*b*d*log(x) - 3*a**2*b*e/x - 3*
a**2*c*d/x - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/x - 3*a*b**2*e/(2*x**2) - 3*a
*b*c*d/x**2 - 2*a*b*c*e/x**3 - a*c**2*d/x**3 - 3*a*c**2*e/(4*x**4) - b**3*d
/(2*x**2) - b**3*e/(3*x**3) - b**2*c*d/x**3 - 3*b**2*c*e/(4*x**4) - 3*b*c**
2*d/(4*x**4) - 3*b*c**2*e/(5*x**5) - c**3*d/(5*x**5) - c**3*e/(6*x**6), Eq(
n, -1)), (a**3*d*x + 2*a**3*e*sqrt(x) + 6*a**2*b*d*sqrt(x) + 3*a**2*b*e*log
(x) + 3*a**2*c*d*log(x) - 6*a**2*c*e/sqrt(x) + 3*a*b**2*d*log(x) - 6*a*b**2
*e/sqrt(x) - 12*a*b*c*d/sqrt(x) - 6*a*b*c*e/x - 3*a*c**2*d/x - 2*a*c**2*e/x
**(3/2) - 2*b**3*d/sqrt(x) - b**3*e/x - 3*b**2*c*d/x - 2*b**2*c*e/x**(3/2)
- 2*b*c**2*d/x**(3/2) - 3*b*c**2*e/(2*x**2) - c**3*d/(2*x**2) - 2*c**3*e/(5
*x**(5/2)), Eq(n, -1/2)), (a**3*d*x + 3*a**3*e*x**(2/3)/2 + 9*a**2*b*d*x**(
2/3)/2 + 9*a**2*b*e*x**(1/3) + 9*a**2*c*d*x**(1/3) + 3*a**2*c*e*log(x) + 9*
a*b**2*d*x**(1/3) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x) - 18*a*b*c*e/x**(1
/3) - 9*a*c**2*d/x**(1/3) - 9*a*c**2*e/(2*x**(2/3)) + b**3*d*log(x) - 3*b**
3*e/x**(1/3) - 9*b**2*c*d/x**(1/3) - 9*b**2*c*e/(2*x**(2/3)) - 9*b*c**2*d/(
2*x**(2/3)) - 3*b*c**2*e/x - c**3*d/x - 3*c**3*e/(4*x**(4/3)), Eq(n, -1/3))
, (a**3*d*x + 4*a**3*e*x**(3/4)/3 + 4*a**2*b*d*x**(3/4) + 6*a**2*b*e*sqrt(x
) + 6*a**2*c*d*sqrt(x) + 12*a**2*c*e*x**(1/4) + 6*a*b**2*d*sqrt(x) + 12*a*b
**2*e*x**(1/4) + 24*a*b*c*d*x**(1/4) + 6*a*b*c*e*log(x) + 3*a*c**2*d*log(x)
 - 12*a*c**2*e/x**(1/4) + 4*b**3*d*x**(1/4) + b**3*e*log(x) + 3*b**2*c*d*lo
g(x) - 12*b**2*c*e/x**(1/4) - 12*b*c**2*d/x**(1/4) - 6*b*c**2*e/sqrt(x) - 2
*c**3*d/sqrt(x) - 4*c**3*e/(3*x**(3/4)), Eq(n, -1/4)), (a**3*d*x + 5*a**3*e
*x**(4/5)/4 + 15*a**2*b*d*x**(4/5)/4 + 5*a**2*b*e*x**(3/5) + 5*a**2*c*d*x**
(3/5) + 15*a**2*c*e*x**(2/5)/2 + 5*a*b**2*d*x**(3/5) + 15*a*b**2*e*x**(2/5)
/2 + 15*a*b*c*d*x**(2/5) + 30*a*b*c*e*x**(1/5) + 15*a*c**2*d*x**(1/5) + 3*a
*c**2*e*log(x) + 5*b**3*d*x**(2/5)/2 + 5*b**3*e*x**(1/5) + 15*b**2*c*d*x**(
1/5) + 3*b**2*c*e*log(x) + 3*b*c**2*d*log(x) - 15*b*c**2*e/x**(1/5) - 5*c**
3*d/x**(1/5) - 5*c**3*e/(2*x**(2/5)), Eq(n, -1/5)), (a**3*d*x + 6*a**3*e*x*
*(5/6)/5 + 18*a**2*b*d*x**(5/6)/5 + 9*a**2*b*e*x**(2/3)/2 + 9*a**2*c*d*x**(
2/3)/2 + 6*a**2*c*e*sqrt(x) + 9*a*b**2*d*x**(2/3)/2 + 6*a*b**2*e*sqrt(x) +
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12*a*b*c*d*sqrt(x) + 18*a*b*c*e*x**(1/3) + 9*a*c**2*d*x**(1/3) + 18*a*c**2*
e*x**(1/6) + 2*b**3*d*sqrt(x) + 3*b**3*e*x**(1/3) + 9*b**2*c*d*x**(1/3) + 1
8*b**2*c*e*x**(1/6) + 18*b*c**2*d*x**(1/6) + 3*b*c**2*e*log(x) + c**3*d*log
(x) - 6*c**3*e/x**(1/6), Eq(n, -1/6)), (a**3*d*x + 7*a**3*e*x**(6/7)/6 + 7*
a**2*b*d*x**(6/7)/2 + 21*a**2*b*e*x**(5/7)/5 + 21*a**2*c*d*x**(5/7)/5 + 21*
a**2*c*e*x**(4/7)/4 + 21*a*b**2*d*x**(5/7)/5 + 21*a*b**2*e*x**(4/7)/4 + 21*
a*b*c*d*x**(4/7)/2 + 14*a*b*c*e*x**(3/7) + 7*a*c**2*d*x**(3/7) + 21*a*c**2*
e*x**(2/7)/2 + 7*b**3*d*x**(4/7)/4 + 7*b**3*e*x**(3/7)/3 + 7*b**2*c*d*x**(3
/7) + 21*b**2*c*e*x**(2/7)/2 + 21*b*c**2*d*x**(2/7)/2 + 21*b*c**2*e*x**(1/7
) + 7*c**3*d*x**(1/7) + c**3*e*log(x), Eq(n, -1/7)), (5040*a**3*d*n**7*x/(5
040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*
n + 1) + 13068*a**3*d*n**6*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n*
*4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 13132*a**3*d*n**5*x/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6769
*a**3*d*n**4*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
 + 322*n**2 + 28*n + 1) + 1960*a**3*d*n**3*x/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 322*a**3*d*n**2*x/(
5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28
*n + 1) + 28*a**3*d*n*x/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + a**3*d*x/(5040*n**7 + 13068*n**6 + 13132
*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5040*a**3*e*n**6*x*x
**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
 + 28*n + 1) + 8028*a**3*e*n**5*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5
 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5104*a**3*e*n**4*x*x**n/(
5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28
*n + 1) + 1665*a**3*e*n**3*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 67
69*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 295*a**3*e*n**2*x*x**n/(5040*n
**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1
) + 27*a**3*e*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + a**3*e*x*x**n/(5040*n**7 + 13068*n**6 + 1
3132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15120*a**2*b*d*n
**6*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 24084*a**2*b*d*n**5*x*x**n/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15312*a**2*b*d*
n**4*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 4995*a**2*b*d*n**3*x*x**n/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 885*a**2*b*d*n*
*2*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 32
2*n**2 + 28*n + 1) + 81*a**2*b*d*n*x*x**n/(5040*n**7 + 13068*n**6 + 13132*n
**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a**2*b*d*x*x**n/(504
0*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
+ 1) + 7560*a**2*b*e*n**6*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15822*a**2*b*e*n**5*x*x**(2
*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
 + 28*n + 1) + 11787*a**2*b*e*n**4*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 4260*a**2*b*e*n**3
*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 810*a**2*b*e*n**2*x*x**(2*n)/(5040*n**7 + 13068*n**6
 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 78*a**2*b*e*
n*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
 322*n**2 + 28*n + 1) + 3*a**2*b*e*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7560*a**2*c*d*n**6
*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 15822*a**2*c*d*n**5*x*x**(2*n)/(5040*n**7 + 13068*n*
*6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 11787*a**2
*c*d*n**4*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 196
0*n**3 + 322*n**2 + 28*n + 1) + 4260*a**2*c*d*n**3*x*x**(2*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 81
0*a**2*c*d*n**2*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4
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 + 1960*n**3 + 322*n**2 + 28*n + 1) + 78*a**2*c*d*n*x*x**(2*n)/(5040*n**7 +
 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3
*a**2*c*d*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 196
0*n**3 + 322*n**2 + 28*n + 1) + 5040*a**2*c*e*n**6*x*x**(3*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 11
388*a**2*c*e*n**5*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n*
*4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 9336*a**2*c*e*n**4*x*x**(3*n)/(5040
*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n +
 1) + 3657*a**2*c*e*n**3*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 741*a**2*c*e*n**2*x*x**(3*n)
/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +
28*n + 1) + 75*a**2*c*e*n*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a**2*c*e*x*x**(3*n)/(5040
*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n +
 1) + 7560*a*b**2*d*n**6*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 15822*a*b**2*d*n**5*x*x**(2*
n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
+ 28*n + 1) + 11787*a*b**2*d*n**4*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 4260*a*b**2*d*n**3*
x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 810*a*b**2*d*n**2*x*x**(2*n)/(5040*n**7 + 13068*n**6
+ 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 78*a*b**2*d*n
*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 3*a*b**2*d*x*x**(2*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5040*a*b**2*e*n**6*
x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 11388*a*b**2*e*n**5*x*x**(3*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 9336*a*b**2
*e*n**4*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*
n**3 + 322*n**2 + 28*n + 1) + 3657*a*b**2*e*n**3*x*x**(3*n)/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 741*
a*b**2*e*n**2*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
 1960*n**3 + 322*n**2 + 28*n + 1) + 75*a*b**2*e*n*x*x**(3*n)/(5040*n**7 + 1
3068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a
*b**2*e*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*
n**3 + 322*n**2 + 28*n + 1) + 10080*a*b*c*d*n**6*x*x**(3*n)/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2277
6*a*b*c*d*n**5*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4
+ 1960*n**3 + 322*n**2 + 28*n + 1) + 18672*a*b*c*d*n**4*x*x**(3*n)/(5040*n*
*7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
 + 7314*a*b*c*d*n**3*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1482*a*b*c*d*n**2*x*x**(3*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
 + 1) + 150*a*b*c*d*n*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 676
9*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6*a*b*c*d*x*x**(3*n)/(5040*n**7
 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) +
 7560*a*b*c*e*n**6*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n
**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 17712*a*b*c*e*n**5*x*x**(4*n)/(504
0*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
+ 1) + 15270*a*b*c*e*n**4*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 +
 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 6336*a*b*c*e*n**3*x*x**(4*n
)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +
 28*n + 1) + 1356*a*b*c*e*n**2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n
**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 144*a*b*c*e*n*x*x**(4*
n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2
+ 28*n + 1) + 6*a*b*c*e*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6
769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3780*a*c**2*d*n**6*x*x**(4*n)
/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 +
28*n + 1) + 8856*a*c**2*d*n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n
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**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7635*a*c**2*d*n**4*x*x
**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 3168*a*c**2*d*n**3*x*x**(4*n)/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 678*a*c**2*d*n*
*2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
+ 322*n**2 + 28*n + 1) + 72*a*c**2*d*n*x*x**(4*n)/(5040*n**7 + 13068*n**6 +
 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*a*c**2*d*x*x
**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 3024*a*c**2*e*n**6*x*x**(5*n)/(5040*n**7 + 13068*n**6 +
13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*a*c**2*e*n
**5*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
 + 322*n**2 + 28*n + 1) + 6432*a*c**2*e*n**4*x*x**(5*n)/(5040*n**7 + 13068*
n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2775*a*c
**2*e*n**3*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 19
60*n**3 + 322*n**2 + 28*n + 1) + 621*a*c**2*e*n**2*x*x**(5*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 69
*a*c**2*e*n*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + 3*a*c**2*e*x*x**(5*n)/(5040*n**7 + 13068*
n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1680*b**
3*d*n**6*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960
*n**3 + 322*n**2 + 28*n + 1) + 3796*b**3*d*n**5*x*x**(3*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3112*
b**3*d*n**4*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1
960*n**3 + 322*n**2 + 28*n + 1) + 1219*b**3*d*n**3*x*x**(3*n)/(5040*n**7 +
13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 24
7*b**3*d*n**2*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
 1960*n**3 + 322*n**2 + 28*n + 1) + 25*b**3*d*n*x*x**(3*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + b**3*
d*x*x**(3*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
 322*n**2 + 28*n + 1) + 1260*b**3*e*n**6*x*x**(4*n)/(5040*n**7 + 13068*n**6
 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2952*b**3*e*
n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + 2545*b**3*e*n**4*x*x**(4*n)/(5040*n**7 + 13068*n
**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1056*b**3
*e*n**3*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*
n**3 + 322*n**2 + 28*n + 1) + 226*b**3*e*n**2*x*x**(4*n)/(5040*n**7 + 13068
*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 24*b**3
*e*n*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + b**3*e*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3780*b**2*c*d*n**6*
x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 3
22*n**2 + 28*n + 1) + 8856*b**2*c*d*n**5*x*x**(4*n)/(5040*n**7 + 13068*n**6
 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7635*b**2*c*
d*n**4*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
**3 + 322*n**2 + 28*n + 1) + 3168*b**2*c*d*n**3*x*x**(4*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 678*b
**2*c*d*n**2*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
1960*n**3 + 322*n**2 + 28*n + 1) + 72*b**2*c*d*n*x*x**(4*n)/(5040*n**7 + 13
068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b*
*2*c*d*x*x**(4*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
**3 + 322*n**2 + 28*n + 1) + 3024*b**2*c*e*n**6*x*x**(5*n)/(5040*n**7 + 130
68*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*
b**2*c*e*n**5*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 +
 1960*n**3 + 322*n**2 + 28*n + 1) + 6432*b**2*c*e*n**4*x*x**(5*n)/(5040*n**
7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
+ 2775*b**2*c*e*n**3*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 621*b**2*c*e*n**2*x*x**(5*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
 + 1) + 69*b**2*c*e*n*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 676
```

```
9*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b**2*c*e*x*x**(5*n)/(5040*n**
7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1)
+ 3024*b*c**2*d*n**6*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769
*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 7236*b*c**2*d*n**5*x*x**(5*n)/(5
040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*
n + 1) + 6432*b*c**2*d*n**4*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5
 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2775*b*c**2*d*n**3*x*x**(
5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 621*b*c**2*d*n**2*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 1313
2*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 69*b*c**2*d*n*x*x**
(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n*
*2 + 28*n + 1) + 3*b*c**2*d*x*x**(5*n)/(5040*n**7 + 13068*n**6 + 13132*n**5
 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2520*b*c**2*e*n**6*x*x**(
6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 6114*b*c**2*e*n**5*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 5547*b*c**2*e*n**4
*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 2460*b*c**2*e*n**3*x*x**(6*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 570*b*c**2*
e*n**2*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n
**3 + 322*n**2 + 28*n + 1) + 66*b*c**2*e*n*x*x**(6*n)/(5040*n**7 + 13068*n*
*6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 3*b*c**2*e
*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
322*n**2 + 28*n + 1) + 840*c**3*d*n**6*x*x**(6*n)/(5040*n**7 + 13068*n**6 +
 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 2038*c**3*d*n*
*5*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3
+ 322*n**2 + 28*n + 1) + 1849*c**3*d*n**4*x*x**(6*n)/(5040*n**7 + 13068*n**
6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 820*c**3*d*
n**3*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**
3 + 322*n**2 + 28*n + 1) + 190*c**3*d*n**2*x*x**(6*n)/(5040*n**7 + 13068*n*
*6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 22*c**3*d*
n*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 +
 322*n**2 + 28*n + 1) + c**3*d*x*x**(6*n)/(5040*n**7 + 13068*n**6 + 13132*n
**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 720*c**3*e*n**6*x*x**(
7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**
2 + 28*n + 1) + 1764*c**3*e*n**5*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 13132
*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 1624*c**3*e*n**4*x*x
**(7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*
n**2 + 28*n + 1) + 735*c**3*e*n**3*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 131
32*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + 175*c**3*e*n**2*x*
x**(7*n)/(5040*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322
*n**2 + 28*n + 1) + 21*c**3*e*n*x*x**(7*n)/(5040*n**7 + 13068*n**6 + 13132*
n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n + 1) + c**3*e*x*x**(7*n)/(50
40*n**7 + 13068*n**6 + 13132*n**5 + 6769*n**4 + 1960*n**3 + 322*n**2 + 28*n
 + 1), True))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```